Essays on business and financial cycles : prediction and synchronization

Essays On
Business and Financial Cycles - Prediction and Synchronization

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Contents

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Chapter 1

Introduction

After a period of '*Great Moderation*', the incidence of '*Great Recession*' that started in 2007-08 has brought the discussion of prediction of real and financial cycles to the fore. Further, the eurozone debt crises and the events that lead to it have also rekindled the question of synchronization of cross-country economic cycles. Furthermore, the depth and breadth of the two events further underscore the need to understand and forecast cycles in order for the policy makers to implement proactive policies. The forecasting may thus provide *early warning signals* to the policy makers and regulators.

Business and financial cycles are an essential feature of economic reality. This area has, therefore, attracted the interest of researchers in both academia and policy circles. From 'Sunspot cycle' view of Jevons (1909) to the 'Investment-led cycles' theory of Austrian and Keynesian schools; from 'Financial Instability Hypothesis' of Minsky (1977) to the Neo-Keynesian 'Income Distribution' theories; from 'Irrational Exuberance' view of Schiller (2000) to the recent behavioral 'Theory of Reflexivity' by Soros (2010), there has been a tremendous amount of interest in the subject. While these theories debate the salient features and assign different causes to the boom-bust cycles, a vast amount of empirical literature also exists that attempts to explain and predict the business and financial cycles. The classical approach of Burns and Mitchell (1946) examines the behavior of various leading, coincident and lagging economic indicators to identify the cyclical periods in the economy. Their approach in essence is similar to that adopted by National Bureau of Economic Research (NBER) for dating the U.S. business cycles. The empirical literature that followed their seminal contribution utilizes a combination of economic indicators in linear and non-linear models to forecast business cycles.

Prediction of financial cycles is an equally important area. Financial markets provide grease in the form of capital and credit for smooth running of the engine of economy. Hence, upheavals in the financial markets have consequences for the real sector of the economy. Further, the technological advances have facilitated the cross-border investment in the stock and currency markets. Therefore, prediction of financial cycles has also been of interest to the investors, market practitioners, and the policy makers. For the later, the sheer amount of bail-out costs of financially distressed institutions have prompted the governments to explicitly or implicitly entrust their central banks with the task of ensuring financial stability. In fact the recent episode of subprime crisis has its roots in the financial market and, due to cross-border exposures, quickly spread worldwide and caused the 'Great Recession'. The prediction of cyclical episodes in the financial markets is, therefore, a burgeoning area of research, (Perez-Quiros and Timmermann, 2000; Coakley and Fuertes, 2006; Chen, 2009, are examples of recent studies).

In this vein, the thesis is a collection of four self-contained essays on business and financial cycles, especially their prediction and bilateral synchronization. These essays can also be seen as empirical applications of binary choice models (BCM), since we mainly apply and extend the BCMs, specifically the probit model, to forecast cycles in real and financial sectors of the economy. For empirical study of financial cycles, we consider two assets classes viz., stocks and currencies. In Chapter 1, we consider the cyclical ups-and-downs, also termed 'bulls' and 'bears', on the stock market. We first extract, parametrically as well as non-parametrically, the bulls (up states) and bears (down states) on the U.S. stock market. The parametric approach extracts the filtered probabilities of bear market via Markov switching model (MSM) of Hamilton (1989). These bear probabilities are then explained via linear predictive regressions using a battery of macro-financial variables. Non-parametrically, we follow the algorithm developed in Bry and Boschan (1971). The algorithm locates the peaks and troughs in the time series based on some censoring criteria. An indicator time series is then formed by marking the periods from peak-to-trough as '1' (bear) and '0' (bull) otherwise. In all the chapters, we extract cycles using Bry and Boschan algorithm, unless otherwise stated. Since the series so obtained are binary in nature, we use the binary choice models - logit and probit - to forecast the probabilities of the market being in a bear state using the same macro-financial variables as in linear regressions. One of the main innovation of our study is to account for the stylized facts of the cycles, i.e., persistent nature of bear conditions and the asymmetries. To this end, we use various extensions of BCMs proposed in Kauppi and Saikkonen (2008). These dynamic extensions consist of using on RHS (as explanatory variables) i) the lagged dependent variable (LDV) (persistence) ii) an interaction of LDV and the key explanatory variable (asymmetry) and iii) a lagged autoregressive term (expectations). It turns out that by including the LDV (lagged stock market phase dummy variable) in the list of explanatory variables, the dynamic BCMs better capture the empirical stylized facts of the stock market cycles. Moving on, the forecasting performance is compared not only across the linear (predictive regressions) and non-linear (BCM) models but also between the three most widely used models within the non-linear category (i.e., Probit, Logit and MSM). Therefore, next to using an eclectic battery of forecast evaluation metrics, we also compare the forecast performance of different nested and non-nested models via tests proposed in Clark and West (2007) and Diebold and Mariano (1995), respectively. We find that BCMs outperform MSM, and, within the BCMs, probit model performs better than the logit specification. Finally, as an economic utility of our exercise, we show that active portfolio management pays-off: an active stock market trading strategy based on predictions from both static as well as dynamic BCMs outperforms the passive one. Particularly, the trading strategy based on dynamic BCM specification with lagged phase dummy fetches higher returns than other BCM specifications.

In Chapter 2, we turn our attention to another financial asset class, viz., the bilateral currency exchange rate. Because this market is one of the largest in terms of trading volume and most of the transactions represent cross-border activities, research in this area has been multifaceted. Various aspects have been explored including theoretically reconciling the exchange rate to macro- and micro-economic variables and forecasting the currency crises. Although it has long been recognized that the exchange rate could be considered as an asset price like any other financial asset, e.g., equity prices (Mussa, 1976; Frenkel and Mussa, 1985); and that there are periods of appreciation (bull) or depreciation (bear) (see, e.g., MacDonald and Young, 1986), the exchange rate cycles remain an under-explored area. The cycles in the exchange rate can be induced by either the demand and supply shocks or the government intervention to stabilize or correct its misalignment. We build on this observation and extract cyclical phases via Bry and Boschan algorithm in the exchange rates of six major currencies vis-a-vis the U.S. dollar. We use cross-country differential of uncovered interest rate parity (UIP), relative purchasing power parity (RPPP), pseudo parity with equity returns, liquidity pressures and term spread to forecast the depreciation probability of a currency via static and dynamic probit models. We find that predictions over a horizon of one to twenty four months based on these five variables, which we term as risk factors, do exhibit cyclical variations. In fact these factors forecast periods of depreciation or appreciation and subsequent reversals. We also find that ignoring the dynamics of exchange rates leads to systematic under/overestimation by agents as the risk factors are generally significant at one to twenty-four month horizons when employing static probit models. However, estimating an information augmented dynamic probit model demonstrates that violations of the UIP, RPPP and the pseudo-parity are, in fact, short-term phenomena. Our proposed framework has practical utility for policy makers to smoothen the currency misalignment and for investors to form trading strategies and hedge their positions as well as rebalance their carry trade positions.

Next, in Chapter 3, we consider the issue of synchronization of business and financial cycles in the Euro area. The current eurozone turmoil has painfully reminded us that too little economic convergence - and more specifically a lack of business cycle synchronization - is problematic for currency unions to be viable. Moreover, real and financial market integration are interrelated because financial market fluctuations reflect the market's expectations about the future real economic activity. In the existing literature, the issue of bilateral business and/or financial cycle synchronization is typically approached within a linear framework. To allow for the possibility of non-linear linkages, we propose a probit based approach and apply it to identify cyclical real and financial synchronization within and between four major eurozone countries (France, Germany, Italy and Spain) and five smaller eurozone countries. Our results suggest that real and financial synchronization, identified within the probit framework, are both economically and statistically significant. Moreover, there is more of financial synchronization than business cycle synchronization within the eurozone, especially after the introduction of the single currency. Our results regarding financial synchronization support the plea for a more uniform implementation of macro-prudential regulations. The Euro introduction seems to have had mixed effects on business cycle synchronization though. Some countries' business cycles like those of Greece, Italy or Spain even seem to have decoupled from the European core after 1999.

In Chapter 2, we recognized and accounted for persistence and asymmetries in the financial cycles by using the dynamic versions of probit model. The serial dependence and volatility clustering, however, leads to conditional heteroskedasticity in the model disturbances (Engle, 1982). The issue becomes even more pronounced in BCMs of time series where aim generally is to make predictions about, e.g., recessions, stock market bears, financial crisis etc. The estimates and inference from these models, therefore, become inconsistent and inefficient. To cope with this, we propose an adjustment for conditional heteroskedasticity in BCMs along the lines of Engle (1982) and Bollerslev (1986) in Chapter 5. Specifically, we propose a GARCH-type adjustment for the conditional variance of the model errors in the short run, while leaving the unconditional (long run) variance fixed to unity to achieve identification. Data augmentation type algorithm is developed to estimate the model via maximum likelihood. Simulation results show that the proposed heteroskedasticity adjusted model leads to reduction in bias of estimates, compared with the unadjusted model. We also propose Lagrange multiplier (LM) tests for testing the ARCH effects in BCMs along the lines of Davidson and MacKinnon (1984). Of the proposed LM tests, one based on the expected Information matrix turns out to have better size and power properties. Empirical applications to predicting the US business and financial cycles using term spread confirm the utility of heteroskedastic adjustment.

Finally, Chapter 6 concludes the thesis with a general conclusion based on the previous chapters.

Chapter 2

Predicting Stock Market Bears in the U.S.[1]

2.1 Introduction

Identifying well-performing approaches towards predicting individual and aggregate stock market behavior constitutes one of the most actively researched areas in financial economics in general (and empirical asset pricing in particular), see e.g.(Campbell et al., 1997, Ch. 2) for a classic overview of financial return predictability tests in general. The predictability debate is also closely related to the debate on whether financial markets are efficient or not, i.e. can *excess* stock returns be predicted using past information?[2] Various lagged instruments - mainly financial and macro-economic indicators like interest rates, inflation, default risk, aggregate output, money stocks, unemployment - have been put at work through time in the quest to identify stock market predictability, see e.g., Pesaran and Timmermann (1995); Rapach et al. (2005); Chen (2009). Other financial variables like size, value, accounting ratios, sales growth, etc. have also been tried, see e.g. Cochrane (2008a). The empirical outcomes from these types of prediction studies are mixed: whereas e.g. Rapach et al. (2005) or Cochrane (2008b), find support for return predictability, others like e.g., Goyal and Welch (2003, 2008) or Chen (2009), do not. A related strand of literature focuses on predicting the direction of the stock returns as it exhibits a larger degree of dependence over time, see e.g., Pesaran and Timmermann

[1]This chapter is based on Candelon, Ahmed and Straetmans (2012)

[2]One should be careful to distinguish return predictability from excess return predictability. The latter requires specifying an equilibrium asset pricing model as benchmark to test against. Thus, the presence of return predictability does not imply that abnormal or excess returns (relative to some equilibrium asset pricing benchmark) are also predictable (or vice versa).

(1994), Christoffersen and Diebold (2006), Anatolyev and Gospordinov (2010) and Nyberg (2011).

This chapter is not meant to deepen the vast (excess) stock return predictability cum market efficiency literature. We rather start from the observation that the predictive ability of most traditional linear factor models to the stock market seems to break down completely during periods of extreme volatility like financial crises. Consequently, one may question the economic value-added of traditional linear affine approaches towards return predictability as they seem unable to predict financial asset prices when trustworthy predictions are most needed. This inability of traditional economic models to predict the economy (and financial markets in particular) has seriously eroded the credibility of economists in the aftermath of the current financial and economic crisis.

Starting from the observation that the bulk of the empirical literature on return prediction is based on linear projections of current returns on lagged instruments including past returns, we question whether alternative non-linear approaches exhibit a value-added from a prediction perspective. Moreover, we question whether the more modest objective of predicting stock market *cycles* is a worthwhile exercise to undertake. More specifically, we focus on the prediction of bull periods (persistent expansions) and bear periods (persistent contractions) for the US stock market. Obviously, predicting stock market bulls and bears necessarily entails a two-stage procedure because one first needs to determine the upward and downward phases that determine the bull and bear periods on the stock market. There is no consensus in the literature on how to determine bull and bear phases. We therefore extract these stock market cycles by means of parametric and non-parametric approaches. The parametric approach implies extracting filtered probabilities of bear states via a two-state Markov-switching model (MSM). As for the non-parametric approach, the popular Bry and Boschan (1971) (BB) dating algorithm is employed that maps the original stock market index into a dummy variable of zeros (bears) and ones (bulls). Next, we run linear predictive regressions for the filtered Markov-switching probabilities as well as non-linear binary choice models (BCM) for the binary variables determined with the BB algorithm, see Kauppi and Saikkonen (2008). We compare the forecasting power of these alternative models (within and across different model classes) by using both nested and non-nested hypothesis tests.

This chapter contributes in several ways to the literature on stock market predictability. First, our focus is on *dynamic* rather than *static* versions of the BCM class of models. By including the lagged stock market phase dummy variable in the list of explanatory variables, dynamic BCM models better capture the empirical stylized facts of the stock market cycles like persistence and asymmetries. To our knowledge this is the first attempt to apply *Dynamic Autoregressive Binary Choice* models to financial data.[3] Second, we employ both probit and logit analysis to investigate the robustness of our forecasting results to the distributional choice that links the explanatory variables and the stock market cycle dummy. The models' forecasting performance is compared across the linear (predictive regressions) and non-linear (BCM and MSM) models but also between the three most widely used models within the non-linear category (i.e., Probit, Logit and MSM). Third, next to using an eclectic battery of forecast evaluation metrics, we also compare the forecast performance of different nested and non-nested models via tests proposed in Clark and West (2007) and Diebold and Mariano (1995), respectively. Fourth, the sample period contains the recent 2007-2009 global stock market downturn which "stress tests" the forecasting ability of the models even further. Finally, we show that active stock market trading using dynamic BCM specifications outperform passive trading on both static non-linear models and linear predictive regressions.

Forecasting stock market bears regimes is of potential interest for both investors and policy makers. By forecasting peaks and troughs, investors can potentially realize abnormal returns by using the bulls and bears prediction framework for portfolio rebalancing decisions. Shen (2003) already established that using static binary choice models to determine buy and sell signals creates abnormal returns as compared to passive trading strategies. We show that adding dynamic terms to the BCM model class can further increase the profitability of portfolio rebalancing strategies as compared to passive strategies. Resnick and Shoesmith (2002) also show that profitable market timing strategies can be based on the home country yield spread. From a policy perspective, and given the importance of financial stability for real economic growth and investments, Borio and Lowe (2002) proposes a policy response to bull mar-

[3] As to date, BCM models have mainly been employed to explain and predict business cycle fluctuations. Estrella and Mishkin (1998) or Estrella and Hardouvelis (1991a) model the business cycle using a static variant of BCM. More recently, Kauppi and Saikkonen (2008) proposed a novel methodology to improve upon the static approach by including the lagged business cycle in the list of explanatory variables. Chen (2009) studied the predictability of the US stock market bears by using static versions of binary response models.

kets and to contain asset price imbalances to maintain financial stability. The central bank, which is also responsible for maintaining and ensuring financial stability, can use the proposed dynamic BCM models class to predict stock market bulls using macro-financial variables as Early Warning Signals (*EWS*).

Anticipating our results we show that dynamic extensions of simple probit and logit models both improve in-sample fit and out-of-sample forecasting power as compared to static binary response models, Markov-switching models and linear predictive regressions. Upon using the Clark and West (2007) test for comparing the predictive ability of nested models, macro-financial variables exhibit a stronger predictive ability for the dynamic BCM class in contrast to the linear, static probit or logit and Markov-switching models where few variables are seen to have predictive ability. Upon comparing non-nested model specifications, the Diebold and Mariano (1995) test statistic does not show a significant difference in forecasting ability between logit and probit analysis, except when asymmetries are introduced in the model. In the latter case, probit models outperform logit models. The Diebold and Mariano (1995) test also reveals that binary choice models (regardless the considered form of dynamics) generally outperform the Markov-switching model. The general picture that emerges by comparing the forecasting performance of nested and non-nested models (using the Diebold and Mariano (1995) test and the Clark and West (2007) test) is that the dynamic BCM class seems to outperform other nested and non-nested models. Finally, we also show that investors re-balancing their portfolios using dynamic BCM specifications realize higher abnormal returns as compared to either the use of static BCM specifications or purely passive trading strategies

The rest of the chapter is organized as follows. Sections 2-4 describe the econometric methodology. Section 2.5 provides a detailed data description and presents empirical results. Section 2.6 discusses potential economic applications of non-linear prediction of stock market cycles. Section 2.7 concludes.

2.2 Dating Bulls and Bears

Prior to predicting bear and bull phases, we need to define what stock market “bulls” and “bears” are. There is no clear consensus - either in the popular press or the academic literature - on what bulls and bears actually mean, see e.g. Candelon et al. (2008a) for an earlier discussion. We use the definition due to Chauvet and Potter (2000a) who describe bulls and bears as periods of generally increasing and decreasing market prices, respectively. From the latter definition it naturally follows that stock prices evolve between local peaks and troughs that mark the boundaries of the bull and bear periods. This concept of persistent rises and falls is in accordance with the largest part of the business cycle literature and goes back to the seminal analysis of Burns and Mitchell (1946). The phase definition above essentially means that a bullish stock market turns bearish if prices fall for a sustained period after their previous (local) peak. Such a characterization does not exclude sustained price falls (rises) during a bull (bear) phase but there are restrictions on the extent to which these sequences of price reversals can occur and yet still be considered part of any given bull or bear phase. Exploiting the above definition of bulls and bears, we implement a parametric and a nonparametric approach for dating the corresponding local peaks and troughs.

2.2.1 Parametric Dating - Markov Switching Model

Our parametric dating approach makes use of the Markov-switching model (MSM), see Hamilton (1989).[4]

Let $r_t = 100 \ln(Y_t/Y_{t-1})$ represent the (log) return of the US stock market index (with Y_t referring to the price level of the market index). The unobserved state variable s_t is a latent dummy variable that either equals 0 (bull market) or 1 (bear market). Assume that stock returns follow a simple two-state mean/variance Markov-switching model: Markov process:

$$r_t = \mu_{s_t} + \varepsilon_t, \qquad \varepsilon_t \sim \text{i.i.d.} \quad \mathcal{N}(0, \sigma^2_{s_t}), \tag{2.1}$$

where mean μ_{s_t} and variance $\sigma^2_{s_t}$ depend on state s_t. The unobserved state variable s_t is a latent dummy variable that can either take the values of 0 (bulls) or 1 (bears). Moreover, it is assumed to be governed by a first-order Markov chain process whose fixed transition probabilities are equal to:

$$P\{s_t = j | s_{t-1} = i\} = p_{ij} \quad \forall i, j = 0, 1. \tag{2.2}$$

In particular, $p_{11} = P\{s_t = 1 | s_{t-1} = 1\}$ denotes the probability of staying in a bear state whereas $p_{00} = P\{s_t = 0 | s_{t-1} = 0\}$ refers to the the probability of staying in a bull state. The parameters and the probabilities are estimated

[4]Hamilton and Lin (1996); Maheu and McCurdy (2000) and Chen (2009) also use this approach in order to identify stock market bull and bear regimes.

via maximum likelihood. For further analysis in the rest of the chapter, we consider filtered probabilities, which represents the inference about the state variable, s_t, given information up to time t, i.e. $Pr(s_t = i|r_t)$.

2.2.2 Non-parametric Approach - Bry-Boschan Algorithm

The non-parametric approach that we implement goes back to the algorithm developed by Bry and Boschan (1971). It was originally developed to determine business cycle phases but it has also been applied to determine cycles in stock market data, see e.g. Edwards et al. (2003a); Pagan and Sossounov (2003); Candelon et al. (2008a) or Chen (2009). This nonparametric dating algorithm recognizes time series patterns, disentangles these patterns according to a sequence of rules before locating the turning points (peaks and troughs) in the series.

Pagan and Sossounov (2003) observe that the nature of financial asset prices is sufficiently different from real quantities so that some modifications in the original Bry-Boschan (BB) algorithm are desirable. In particular, we do not smooth the series as this will entail information loss (mainly the elimination of outliers that constitute essential features of bull and bear periods).[5]

Loosely speaking, the location of turning points within the BB algorithm amounts to identifying local maxima or minima within a window of six months. More specifically, a turning point represents a peak at time t if, $y_{t-3}, ..., y_{t-1} < y_t > y_{t+1}, ..., y_{t+3}$ whereas it represents a trough if, $y_{t-3}, ..., y_{t-1} > y_t < y_{t+1}, ..., y_{t+3}$, with $y_t = \log(Y_t)$ standing for the (log) of the stock market index. Finally, periods from peak to trough are classified as a *bear* $(S_t = 1)$ while those from trough to peak are classified as a *bull* $(S_t = 0)$.

2.3 Models to predict bears

We distinguish between linear predictive regressions and non-linear binary choice models. Within the latter model class we further distinguish between static and dynamic models. Dynamics are introduced by including the (lagged) stock market cycle dummy in the list of independent variables at the right hand side of the binary choice regressions.

[5] Additional censoring criteria include: setting a search window for a bear or bull period at six months; ensuring that the complete cycle lasts for sixteen months with alternating peaks and troughs; and eliminating peaks or troughs within three months of beginning and end of the series (see Candelon et al. (2008a) and Pagan and Sossounov (2003)).

2.3.1 Linear Predictive Regressions

After obtaining the filtered probabilities, $P_t(s_t = i) \forall i = 0, 1$, from the Markov-switching model (MSM) in (2.1)., we consider the following predictive regression model:

$$P_t(s_t = 1) = \alpha + \beta x_{t-k} + u_t \qquad \forall u_t \sim \mathcal{N}(0, \sigma^2), \tag{2.3}$$

where x_t is a macrovariable that may potentially predict bear market probability, $P_t(s_t = 1)$.

2.3.2 Binary Choice Models

Binary choice models assume that bulls and bears on the stock market can be modeled by a binary S_t variable, i.e. the market is in either a bull ($S_t = 0$) or a bear ($S_t = 1$) state. We start by introducing the static version of a probit/logit regression.

Such a model is defined by starting from a theoretical linear relationship of the form:

$$S_t^* = \alpha + \beta x_{t-k} + \varepsilon_t, \tag{2.4}$$

where S_t^* is an unobserved variable that determines the occurrence of a stock market bear at time t, k is the length of the forecast horizon and x_{t-k} is a macro/financial variable. The observable bear indicator S_t is related to this model by

$$S_t = \begin{cases} 1 & \text{if} \quad S_t^* > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2.5}$$

We are interested in predicting the probability of bear conditions in the US stock market, given the information about the macro-variable x_{t-k} and the parameter set $\boldsymbol{\theta} = (\alpha, \beta)$ i.e.,

$$P(S_t = 1 | x_t; \boldsymbol{\theta}) = \boldsymbol{F}(\boldsymbol{x_{t-k}}, \boldsymbol{\theta}), \tag{2.6}$$

where $\boldsymbol{F}(.)$ is a cumulative distribution function to be specified. There are many choices for $\boldsymbol{F}(.)$, but we shall consider the two most commonly used, i.e., the standard normal distribution function, $\Phi(z)$, and the logistic distribution function, $\Lambda(z)$. The former assumption leads to the probit model and the latter to the logit model. We employ both distributional assumptions in our empirical application as a robustness check for the empirical outcomes.

If we let $\boldsymbol{\pi}_t = \alpha + \beta x_{t-k}$, and p_t denotes the probability of observing a bear state ($S_t = 1$) at time t, then S_t follows a Binomial distribution with conditional expected value equal to p_t. Formally, let $\mathfrak{F}_t = \sigma\{(S_s, x_s), s \leq t\}$ be the information set available at time t. Then, $S_t | \mathfrak{F}_{t-1} \sim \mathcal{B}(p_t)$, conditioned on $\mathfrak{F}_{t-1}$. Assuming that $\boldsymbol{\pi}_t = \boldsymbol{F}^{-1}(p_t)$,

$$E_{t-1}(S_t) = P_{t-1}(\pi_t > \varepsilon_t) = P_{t-1}(S_t = 1) = \boldsymbol{F}(\boldsymbol{\pi}_t) = p_t. \tag{2.7}$$

Dynamics can be introduced in the process $\boldsymbol{\pi}_t$ in several ways. The dynamic generalizations we present all go back to (Kauppi and Saikkonen (2008)) and are nested into each other. The most obvious dynamic extension requires adding lagged values of S_t to (2.4). The economic justification immediately follows from the stylized fact that stock prices are characterized by sustained rises or falls, see e.g. Guidolin and Timmermann (2005) and Candelon et al. (2008a) for a further discussion. Thus, it should also not come as a surprise that state variables S_t inherit this persistence property. Therefore, adding lagged values of S_t to eq. (2.4) makes sense:

$$\boldsymbol{\pi}_t = \alpha + \sum_{j=1}^{k} x_{t-j}\beta_j + \sum_{j=1}^{q} S_{t-j}\delta_j. \tag{2.8}$$

The dynamics in (2.8) can be further refined by also adding lagged values of $\boldsymbol{\pi}_t$:

$$\boldsymbol{\pi}_t = \alpha + \sum_{j=1}^{k} x_{t-j}\beta_j + \sum_{j=1}^{p} \pi_{t-j}\gamma_j. \tag{2.9}$$

Yet another dynamic extension may be based on the idea that the impact of the explanatory variables may interact with lagged values of the binary variable S_t. For instance, the impact of macro/financial variables like term spread, money growth or inflation may very well differ (asymmetry) during bear phases as compared to bull phases which justifies the inclusion of an interaction term between the fundamental variables x and the cycle dummy S: [6]

$$\boldsymbol{\pi}_t = \alpha + \sum_{j=1}^{k} x_{t-j}\beta_j + S_{t-1}.x_{t-1}\xi. \tag{2.10}$$

[6]Comparable asymmetric impacts have been observed for macro/financial fundamentals across the business cycle, see eg. Neftci (1984); Hamilton (1989).

As a final extension, including an interaction term of p_{t-1} with x_{t-1} (rather than S_{t-1}) may also be useful.

Parameter estimates of all the nested dynamic specifications above can be obtained by using maximum likelihood (ML) optimization.[7] Let $\Theta = (\alpha, \beta, \delta, \gamma, \xi)$ be the parameter vector to be estimated. The relevant (log) likelihood function boils down to:

$$l(\Theta|\mathbf{S}) = \sum_{t=1}^{T} (S_t \ln \boldsymbol{F}(\pi_t(\Theta)) + (1 - S_t) \ln[1 - \boldsymbol{F}(\pi_t(\Theta))]), \tag{2.11}$$

where $\pi_t(\Theta)$ is given by right hand side of (2.4), (2.8), (2.9) or (2.10). In case specifications (2.9) is used, π_t needs to be initialized. Following Kauppi and Saikkonen (2008), we use the unconditional mean of $\boldsymbol{\pi}_t$ as initial value, i.e. $k = p = 1$, $\pi_0 = (\beta_1 \bar{x})/(1 - \gamma_1)$.

2.3.3 Markov-switching Regressions

Apart from extracting the filtered probabilities via (2.1)-(2.2), it is also possible to estimate and filter out the state probabilities via a Markov-switching regression:

$$r_t = \alpha_{s_t} + \beta_{s_t} x_t + \varepsilon_t, \qquad \varepsilon_t \sim \boldsymbol{i.i.d.} \quad \mathcal{N}(0, \sigma_{s_t}). \tag{2.12}$$

where the estimation of the state probabilities is also potentially influenced by the exogenous regressors x_t via the regime-dependent slope parameter β_{s_t}.The estimation is carried out conform section (2.2.1), except that (μ_{s_t}) is now replaced by the right hand side of (2.12) and $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \beta_0, \beta_1, \sigma_0, \sigma_1, p_{00}, p_{11})$. We use this specification to extract bear probabilities and compare these with the probabilities generated by the binary choice models (BCM), described in section (2.3.2). Given that the forecasted bear probabilities from the BCM model class span multiple horizons, we have to construct multi-period probabilities from model (2.12) as well for sake of comparability. We achieve this by calculating the probability of observing at least one bear market over the next k periods, see Candelon, Dumitrescu and Hurlin (2012).

$$\begin{aligned} P(s_{t+1 \ldots t+k} = 1|z_t) &= 1 - P(s_{t+1 \ldots t+k} = 0|z_t) \\ &= 1 - [(p_{10} p_{00}^{k-1} P(s_t = 1|z_t)) + (p_{00}^{k} P(s_t = 0|z_t))]. \end{aligned} \tag{2.13}$$

[7] Chauvet and Potter (2001) propose a Bayesian approach as alternative.

2.4 Evaluating the Models

2.4.1 In-sample fit vs. out-of-sample performance

We are interested in evaluating the considered models' in-sample explanatory power as well as their out-of-sample forecasting power. The explanatory power is traditionally evaluated by looking at the individual coefficient estimates' significance as well as the overall goodness of fit. The individual parameter significance in linear models is evaluated by means of conventional t-statistics; while for binary choice and Markov-switching regression models, Wald-tests and $z-$ tests are employed.[8] The goodness of fit is evaluated by means of the conventional R^2 for linear models and the *pseudo-* R^2 for binary models, see Estrella (1998). Furthermore, for non-linear models, we also look at the likelihood value and the Akaike and Bayesian Information Criteria (*AIC & BIC*) to evaluate the in-sample explanatory power, see Greene (2008).

We also evaluate out-of-sample predictability. For linear models we use the Clark and West (2007) test. As for non-linear models, we use a battery of forecasting diagnostics including the Quadratic Probability Score (QPS), Log Probability Score (LPS), Kuiper's Score (KS), Pietra Index (PI), Bayesian Error Rate (ER) and Area Under ROC (Receiver Operating Curve) (AUC).[9]

2.4.2 Forecast comparison of models

In order to compare the forecasting power of *nested* models, we use the Clark and West (2007) testing procedure that uses the Mean Squared Prediction Error (MSPE) as input. Loosely speaking this test statistic compares whether the difference between an unrestricted model's MSPE and a restricted model's MSPE - nested in the unrestricted model - are significantly different from each other or not. To illustrate the idea, consider a restricted (nested) and an unrestricted version of model (2.3):

$$P_t(s_{t+k} = 1) = \alpha_1 + e_{1,t+k}, \tag{2.14}$$

$$P_t(s_{t+k} = 1) = \alpha_2 + \beta x_{t-k} + e_{2,t+k}. \tag{2.15}$$

[8] A heteroskedasticity and auto-correlation consistent (HAC) robust covariance estimator for parameters is employed to perform the Wald test.

[9] Higher values of KS, AUC and PI and lower values of QPS, LPS and ER imply a better forecasting power, see Candelon, Dumitrescu and Hurlin (2012) for a more detailed discussion of these metrics.

It is obvious that the unrestricted model nests the restricted model under the null hypothesis of a zero slope ($\beta = 0$). Next, we split the total sample of T data points into Q in-sample observations and R out-of-sample observations ($T = Q + R$). A recursive estimation scheme is used for calculating the forecasts. Let $\hat{P}^1_{1,t+k}$ and $\hat{P}^2_{1,t+k}$ be the k-step ahead forecasts from models (2.14) and (2.15) with corresponding forecast errors of $\hat{e}_{1,t+k}$ and $\hat{e}_{2,t+k}$, respectively. Let $\hat{f}_{t+k} = \hat{e}^2_{1,t+k} - [\hat{e}^2_{2,t+k} - (\hat{P}^1_{1,t+k} - \hat{P}^2_{1,t+k})^2]$ be the adjusted MSPE with corresponding sample average $\bar{f} = R^{-1} \sum_{t=R}^{T} \hat{f}_{t+k}$. Moreover, let V_{MSPE} stand for the sample variance of $\hat{f}_{t+k}$. The Clark and West (2007) test statistic now boils down to:

$$\text{MSPE-adj} = \sqrt{R}\bar{f} / \sqrt{V_{MSPE}}. \tag{2.16}$$

The Clark-West statistic tests for equal predictive accuracy in nested models. The null hypothesis of equal predictive accuracy implies that MSPE's should be equal across different nested models while the (less parsimonious) unrestricted model is expected to have a lower MSPE-adj than the restricted model under the alternative hypothesis, i.e. macro/financial factors like x_t exhibit predictive power. The null hypothesis is rejected if the test statistic is sufficiently positive. The test statistic converges in distribution to a standard normal distribution.

We also perform the Clark and West (2007) test within the class of probit and logit models where the restricted model is the static model (2.6) whose forecasting power we want to compare with the dynamic specifications (2.8), (2.9) and (2.10).

Diebold and Mariano (1995) proposes a test for equal predictive accuracy of *non-nested* models. For example, comparing the predictive accuracy of logit model and probit model (either static or dynamic) requires non-nested hypothesis testing but many other pairs may be considered for that purpose within the universe of models we consider in the chapter. Let $e_{i,t+k}$ and $e_{j,t+k}$ be the forecast errors corresponding with two model forecasts $\hat{y}_{i,t+k}$ and $\hat{y}_{j,t+k}$ for y_{t+k}. Moreover, define loss functions corresponding to the forecasts by $g(e_{i,t+k})$ and $g(e_{j,t+k})$, respectively. Let $d_{t+k} = g(e_{i,t+k}) - g(e_{j,t+k})$ be the loss differential that we assume to be co-variance stationary, short memory and asymptotically normally distributed. The null hypothesis of equal predictive accuracy implies that $E[d_{t+k}] = 0$. The Diebold-Mariano (DM) test statistic, which has asymptotic standard normal distribution, is now given by:

$$DM_1 = \frac{\bar{d}}{\sqrt{2\pi \hat{f}_d(0)/T}}, \tag{2.17}$$

where $\bar{d}$ is the mean loss differential and the denominator denotes the standard deviation of d, with $\hat{f}_d(0)$ being the estimate of the spectral density at frequency 0, i.e., $2\pi \hat{f}_d(0) = \sum_{\tau=-(T-1)}^{T-1} 1(\frac{\tau}{S(T)})\hat{\gamma}_d(\tau)$, where $\hat{\gamma}_d(\tau) = (1/T)\sum_{t=|\tau|+1}^{T}(d_t - \bar{d})(d_{t-|\tau|} - \bar{d})$ and $S(T)$ is the truncation lag, see Newey and West (1987).[10] Upon selecting the loss function $g(.)$ to be quadratic, we perform pair-wise tests for different specifications of Logit and Probit models as well as with the Markov-switching regression (2.12) and for multiple forecast horizons.

2.5 Empirical Results

2.5.1 Data

We consider the stock market data and macro-financial variables on the monthly frequency over the period 1957:M1 to 2011:M12. We downloaded the S&P 500 index from Thomson Datastream (clean price index).

Macro-financial variables are obtained from Federal Reserve Economic Data (FRED-II) via the Federal Reserve Bank of St. Louis website.[11] We consider: 5-year and 10-year year term spreads, inflation (change in log-CPI), narrow money growth (M1), broad money growth (M2), growth rate of industrial production , changes the Federal Funds rate, unemployment rate changes and nominal effective exchange rate change.[12]

2.5.2 Linear Predictive Regression Models

As a benchmark for the non-linear prediction model outcomes, we first present empirical findings in Table 2.1 for the linear predictive regression model (2.3) that links filtered Markov-switching probabilities to macro factors. Results for the MS model (2.1)-(2.2) that forms the basis for filtered bear probabilities $P_{1,t+k}$ are reported in panel A of Table 2.1. We also report a simple regression on the

[10] However, rather than letting the truncation lag depend on the sample size, we let it correspond to the forecast horizon as suggested by Diebold and Mariano (1995).

[11] `http://research.stlouisfed.org/fred2/`

[12] Monetary aggregates and exchange rates have different sample periods as compared to the other variables. We downloaded M1 and M2 and the exchange rate over the periods 1959:M1-2011:M12 and 1975:M1-2011:M12, respectively.

mean return at the left hand side of the panel. Non-surprisingly, allowing for regimes strongly increases the (log) likelihood function. As would be expected, mean returns are low in bear periods and high in bull periods. The volatility increases during stock market downturns and decreases during the upturns, which confirms e.g. Maheu and McCurdy (2000); Cunado et al. (2008). Notice also that bulls lasts longer than bears, see Gonzalez et al. (2005); Chen (2009) for earlier references. On average, bulls persist for $1/(1-0.942) = 17$ months whereas a bears are expected to last only $1/(1-0.845)] = 7$ months.

Table 2.1: Results from Linear and Markov-switching models

Panel A: Linear and Markov-switching models

Linear model: $r_t = \mu + \varepsilon_t$			Markov-switching model: $r_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t$						
μ	σ	loglik	μ_1	μ_0	σ_1	σ_0	p_{11}	p_{00}	Loglik
0.506	2.076	-1898	-0.989	1.064	6.183	3.175	0.845	0.942	-1255
(3.02)	(24.28)		(-1.08)	(5.06)	(4.94)	(6.07)	(10.20)	(36.98)	

Panel B: Predictive Regressions - In- and out-of- sample results

k	$\hat{\beta}$	p-val	R^2	CW	$\hat{\beta}$	p-val	R^2	CW
	Term Spread - 10Y				Term Spread - 5Y			
1	0.008	0.353	0.001	**0.003**	0.018	0.093	0.004	**0.009**
3	0.014	0.088	0.004	**0.000**	0.023	**0.030**	0.007	**0.004**
12	0.046	**0.000**	0.046	**0.000**	0.066	**0.000**	0.059	**0.000**
	Inflation				Ind. Prod. - Growth			
1	0.171	**0.000**	0.027	**0.000**	-0.070	**0.000**	0.053	**0.000**
3	0.201	**0.000**	0.038	**0.000**	-0.053	**0.000**	0.030	**0.000**
12	0.110	**0.006**	0.011	**0.005**	-0.001	0.910	0.000	**0.028**
	Nar Money - Growth				Brd Money - Growth			
1	0.038	**0.015**	0.009	0.059	0.045	0.129	0.004	**0.003**
3	0.022	0.157	0.003	0.051	0.015	0.606	0.000	**0.001**
12	0.003	0.845	0.000	0.911	0.012	0.691	0.000	0.121
	Unemp. Rate - Change				Fed. F Rate - Change			
1	0.351	**0.000**	0.064	**0.000**	-0.048	**0.012**	0.010	**0.003**
3	0.284	**0.000**	0.042	**0.000**	-0.017	0.375	0.001	**0.038**
12	0.048	0.375	0.001	**0.010**	-0.014	0.476	0.001	**0.092**
	Exch. Rate - Change							
1	0.000	0.953	0.000	0.182				
3	-0.001	0.938	0.000	0.234				
12	-0.003	0.602	0.001	0.988				

Notes: 1. Panel A: t-statistic in parenthesis. 2. Panel B: Predictive regression, $P_t(s_t = 1) = \alpha + \beta x_{t-k} + u_t$, where $P_t(s_t = 1)$ are the filtered bear probabilities from Markov-switching model. 3. CW is the Clark-West (2007) test for forecast equality of restricted, $P_t(s_t = 1) = \alpha_1 + e_t$, and unrestricted, $P_t(s_t = 1) = \alpha_2 + \beta x_{t-k} + e_t$, models, respectively. 4. **Bold entries** indicate significance at 5% or below level. 6. All estimation results for period 1957:M1-2011M12 except for M1 & M2 (1959:M1-2011:M12) and Exchange Rates (1975:M1-2011:M12).

Using the MS model (2.1)-(2.2), we filter the bear state probabilities using the information available at that time. Figure 2.1 shows these probabilities. There seem to be quite a few spikes. If a threshold of 50% is used, market turmoil as reflected by sharp drops in the S&P 500 index, is captured quite well.[13]

[13]However, the signals transmitted by these probabilities are not fully synchronous to the actual

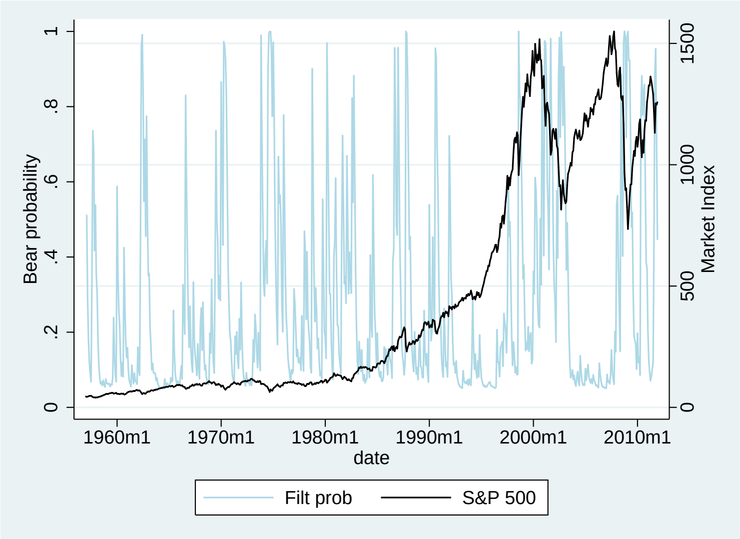

Figure 2.1: Filtered Probabilities for Bear Market from Markov Switching Model (right scale) and S&P 500 Index (left scale) - Sample period 1957:M1-2011M12

Next, we regress these filtered bear probabilities on macro-financial variables via predictive regression (2.3) to investigate the explanatory and predictive power of the latter variables towards bear market conditions. With an eye towards preventing over-fitting and multicollinearity issues, we include each variable separately. Panel B of table 2.1 reports the results and for prediction horizons k equal to of 1, 3 and 12 months.[14] For the sake of conciseness, we arbitrarily term horizons 1-3 as *short*, 6 as *medium* and 12-24 as *long*.

Upon looking at the t-statistics of the $\widehat{\beta}$ slope estimates, only inflation turn out as a significant predictor at all horizons with mixed results for the other variables. As for narrow money growth and exchange rate changes, they do not exhibit explanatory power whatsoever. The goodness-of-fit R^2 is also hardly impressive - reaching a maximum value at 6%. This seems to be in line with the results by Rapach et al. (2005) who also find that inflation is significant over all horizons.

In order to test the *out-of-sample* predictability of macro-financial variables in (2.3), we conduct the Clark and West (2007) test for nested models. In our

drops observed in the index.

[14]We actually estimated the predictive regression, as well as subsequent models, for 1, 3, 6, 12 and 24 month horizons. However, for sake of space considerations, we only report results for 1, 3 and 12 horizons. Other results are available upon request from the authors.

case, we compare the forecasting power of the restricted model (2.14) with the unrestricted model (2.15). The column captioned *CW* in Panel B of table 2.1 provides the results. Nearly all the variables exhibit forecasting power for bear conditions over nearly all considered horizons. Narrow money growth and exchange rate changes constitute exceptions.

2.5.3 Binary Choice Models

In the previous section we observed that although some variables could explain some regression variation, however, the overall fit - in terms of R^2- of the linear predictive regression framework is quite low. In this section, we estimate non-linear binary choice models (BCM) in order to see whether the non-linearity and/or the inclusion of dynamics helps to improve the explanatory and forecasting power. Estimation of the binary choice models (BCM), requires a series of bull and bear periods in the form of an indicator variable ($S_t = 0, 1$). We achieve this by applying the Bry and Boschan (1971) algorithm to the monthly S&P 500 quotes over the sample period January 1957 to December 2011. Once we identified the turning points in this way (peaks and troughs), we can label periods from peak to trough as *bears* ($S_t = 1$) and periods from trough to peak as *bulls* ($S_t = 0$) in order to get a binary time series indicating bulls and bears. Figure 2.2 plots this indicator variable alongside the S&P 500 index. Our identified bear phases largely conform to those obtained by Pagan and Sossounov (2003) and Gonzalez et al. (2005) over their respective periods of estimation.

The BCMs are in essence non-linear models that are expected to reveal non-linearities in the data (if present) which the simple linear model could not have detected. We study in-sample fit and out-of-sample forecasting power for the nested specifications (2.8), (2.9) and (2.10) and/or combinations thereof. Our benchmark model is the simple binary model, $P_{t-k}(S_t = 1) = \boldsymbol{F}(\boldsymbol{\pi}_t)$, where $\boldsymbol{\pi}_t$ is of the form (2.18) with x_{t-k} denoting the macro-financial variable used to forecast k periods ahead:

$$\pi_t = \alpha + x_{t-k}\beta \tag{2.18}$$

Tables 2.2, 2.3, 2.4 and 2.5 display the in-sample and out-of-sample results for static BCM equations and their dynamic counterparts in (2.8), (2.9) and (2.10), respectively. All four tables exhibit the same structure. In-sample and out-of sample results are contained in a left panel and a right panel, respectively. The in-sample results are the coefficient estimates $\widehat{\theta}$ for the included

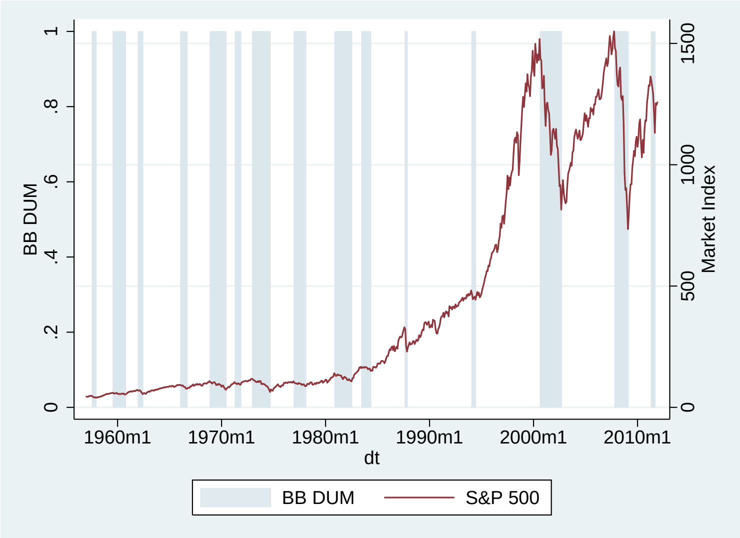

Figure 2.2: Bear Market Indicators as determined via Bry-Boschan Algorithm (left scale) and S&P 500 Index (right scale) - Sample period 1957:M1-2011M12

macro-financial indicator and the dynamic terms, the *Wald test* $p-$value that $\theta = 0$, the pseudo-R^2 and the AIC and BIC information criteria. The out-of-sample results include a whole set of forecasting diagnostics like QPS, LPS, KS, PI, ER and AUC.[15] In-sample and out-of-sample results are generated for forecast horizons of 1, 3 and 12 months. Finally, we only report results for different probit model specifications. Later on in the chapter, we argue that probit outcomes only marginally differ from logit results with the same dynamic specification; but we postpone this non-nested hypothesis testing to a later section.

Table 2.2: Probit: In- and out-of-sample: Model $P_{t-k}(S_t = 1) = \Phi(\alpha + \beta x_{t-k})$

		In-sample			Out-of-sample					
k	$\hat{\beta}$	R^2	**AIC**	**BIC**	**QPS**	**LPS**	**KS**	**PI**	**ER**	**AUC**
	Term Spread - 10Y									
1	0.153	0.020	381.8	386.2	0.388	0.576	0.006	0.044	0.269	0.571
	0.000									
3	0.169	0.024	379.7	384.2	0.387	0.575	0.015	0.057	0.263	0.590
	0.000									
12	0.210	0.036	368.2	372.7	0.379	0.565	0.040	0.048	0.256	0.597
	0.000									

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[15] We deliberately renege from presenting pseudo-R^2 values for the out-of-sample predictions because the latter measure can become negative, see also (Estrella and Mishkin, 1998, p. 47) for a discussion on out-of-sample measures of goodness of fit.

		In-sample			Out-of-sample					
k	$\hat{\beta}$	R^2	AIC	BIC	QPS	LPS	KS	PI	ER	AUC
	Term Spread - 5Y									
1	0.160	0.014	383.9	388.4	0.391	0.580	0.000	0.034	0.269	0.560
	0.004									
3	0.192	0.019	381.4	385.9	0.390	0.577	0.011	0.047	0.268	0.571
	0.000									
12	0.249	0.033	369.4	373.9	0.380	0.567	0.040	0.056	0.258	0.598
	0.000									
	Inflation									
1	0.686	0.018	382.2	386.7	0.390	0.578	0.012	0.059	0.272	0.581
	0.001									
3	0.557	0.012	383.6	388.1	0.393	0.582	-0.004	0.046	0.274	0.579
	0.006									
12	0.409	0.006	377.6	382.0	0.392	0.580	0.000	0.022	0.269	0.525
	0.045									
	Ind. Prod. - Growth									
1	-0.171	0.013	383.8	388.3	0.391	0.580	0.007	0.056	0.272	0.573
	0.014									
3	-0.037	0.001	387.2	391.7	0.398	0.587	0.000	0.001	0.274	0.498
	0.560									
12	0.092	0.004	378.5	383.0	0.393	0.582	0.000	0.027	0.270	0.544
	0.140									
	Nar. Money - Growth									
1	0.075	0.002	375.0	379.5	0.399	0.588	0.000	0.008	0.270	0.501
	0.351									
3	0.030	0.000	374.8	379.3	0.400	0.590	0.000	0.002	0.277	0.500
	0.689									
12	0.149	0.005	364.5	369.0	0.393	0.582	0.004	0.032	0.271	0.556
	0.076									
	Brd. Money - Growth									
1	0.255	0.005	374.0	378.5	0.398	0.587	0.000	0.021	0.273	0.519
	0.087									
3	0.210	0.003	373.9	378.3	0.399	0.588	0.000	0.019	0.277	0.520
	0.161									
12	0.815	0.044	352.4	356.8	0.377	0.562	0.019	0.059	0.250	0.629
	0.000									
	Unemp. Rate - Change.									
1	0.427	0.004	386.8	391.3	0.396	0.585	0.000	0.027	0.274	0.535
	0.135									
3	0.073	0.000	387.4	391.9	0.398	0.588	0.000	0.000	0.274	0.500
	0.809									
12	-0.262	0.001	379.2	383.7	0.394	0.583	0.000	0.003	0.270	0.506
	0.252									
	Fed. F Rate - Change.									
1	0.132	0.003	387.2	391.6	0.396	0.585	0.000	0.025	0.264	0.516

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Table2.2 *Contd. from pre-page*

	0.227									
3	0.188	0.006	385.6	390.1	0.395	0.585	0.000	0.033	0.265	0.541
	0.106									
12	0.107	0.002	378.9	383.4	0.393	0.583	0.000	0.018	0.263	0.520
	0.401									
	Exch. Rate - Chg.									
1	0.058	0.006	235.7	239.8	0.345	0.529	0.000	0.028	0.224	0.536
	0.097									
3	0.080	0.012	234.1	238.1	0.345	0.527	0.000	0.034	0.225	0.572
	0.020									
12	0.041	0.003	233.6	237.7	0.353	0.537	0.000	0.024	0.230	0.524
	0.219									

Table2.2 *concluded*

Note: 1. Wald Test p -value below $\hat{\Theta}$. 2. **Bold** entries show significance at 5% or below. 3. Ranges of different statistics: R^2 = [0,1], LPS = [0,∞), QPS = [0,2], KS = [0,1], Pietra Index (PI) = [-0.354,0.354], ER (Error Rate) = [0,1] and AUC (Area Under ROC Curve) = [0.5,1). 4. KS, AUC and PI have positive orientation meaning that a higher value implies a better fit, while QPS, LPS and ER are negatively oriented. 5. Estimation period: 1957:M1-2011M12 *except* M1 & M2 (1959:M1-2011:M12) and NEER (1975:M1-2011:M12).

The empirical results for the static probit model (2.18) in table 2.2 differ somewhat from the linear regression model (2.3) in table 2.1. Both inflation and the two term spreads are now statistically significant for all horizons. Additionally, industrial production growth and changes in the exchange rate exhibit some significance on short horizons whereas broad money supply growth (M2) and changes in unemployment rate have some explanatory power for medium and longer horizons, respectively. The overall in-sample fit, as measured by the $pseudo - R^2$, is still quite low for all models (reaching a maximum of 4%). The out-of-sample statistics go in the same direction as the in-sample fit. For example, KS is close to zero and AUC is hardly over 0.5.

Table 2.3: Probit: In- and out-of-sample: Model $P_{t-k}(S_t = 1) = \Phi(\alpha + \beta x_{t-k} + \delta S_{t-1})$

	$\hat{\Theta}$		In-sample			Out-of-sample					
k	x_{t-k}	$\S_{t-1}$	R^2	**AIC**	**BIC**	**QPS**	**LPS**	**KS**	**PI**	**ER**	**AUC**
	Term Spread - 10Y										
1	0.120	3.240	0.754	119.9	126.6	0.085	0.177	0.885	0.313	0.046	0.946
	0.082	**0.000**									
3	0.087	3.227	0.752	120.5	127.3	0.086	0.179	0.885	0.313	0.046	0.944
	0.160	**0.000**									
12	0.047	3.236	0.753	117.1	123.8	0.084	0.176	0.888	0.314	0.045	0.943
	0.541	**0.000**									
	Term Spread - 5Y										
1	0.148	3.248	0.754	120.0	126.7	0.085	0.178	0.885	0.313	0.046	0.946
	0.084	**0.000**									
3	0.114	3.232	0.752	120.5	127.2	0.086	0.179	0.885	0.313	0.046	0.943
	0.120	**0.000**									
12	0.034	3.244	0.752	117.2	124.0	0.084	0.176	0.888	0.314	0.045	0.944
	0.708	**0.000**									
	Inflation										
1	0.134	3.235	0.751	121.2	128.0	0.086	0.180	0.885	0.313	0.046	0.943
	0.696	**0.000**									
3	0.123	3.235	0.751	121.2	127.9	0.086	0.180	0.885	0.313	0.046	0.943

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Table2.3 *Contd. from pre-page*

	0.692	**0.000**									
12	0.393	3.293	0.760	114.1	120.8	0.082	0.172	0.890	0.315	0.043	0.949
	0.275	**0.000**									
	Ind. Prod. - Growth										
1	0.036	3.255	0.751	121.2	128.0	0.086	0.180	0.885	0.313	0.046	0.942
	0.719	**0.000**									
3	0.158	3.292	0.754	120.0	126.7	0.086	0.178	0.885	0.313	0.046	0.945
	0.102	**0.000**									
12	0.065	3.288	0.759	114.6	121.3	0.082	0.173	0.890	0.315	0.043	0.945
	0.509	**0.000**									
	Nar. Money - Growth										
1	0.230	3.316	0.763	113.1	119.8	0.083	0.174	0.889	0.314	0.044	0.946
	0.036	**0.000**									
3	-0.109	3.294	0.761	113.9	120.6	0.083	0.175	0.889	0.315	0.043	0.946
	0.410	**0.000**									
12	0.053	3.286	0.760	110.3	116.9	0.082	0.172	0.892	0.316	0.042	0.947
	0.720	**0.000**									
	Brd. Money - Growth										
1	0.508	3.321	0.765	112.4	119.0	0.083	0.172	0.889	0.314	0.044	0.949
	0.023	**0.000**									
3	0.023	3.275	0.760	114.4	121.0	0.084	0.176	0.889	0.314	0.044	0.945
	0.892	**0.000**									
12	0.453	3.260	0.764	108.9	115.5	0.081	0.170	0.892	0.315	0.043	0.947
	0.069	**0.000**									
	Unemp. Rate - Change.										
1	-0.355	3.265	0.751	121.0	127.7	0.086	0.179	0.885	0.313	0.046	0.945
	0.334	**0.000**									
3	-0.560	3.267	0.752	120.6	127.3	0.086	0.179	0.885	0.313	0.046	0.941
	0.174	**0.000**									
12	-0.342	3.294	0.759	114.6	121.3	0.082	0.172	0.890	0.315	0.043	0.945
	0.437	**0.000**									
	Fed. F Rate - Change										
1	0.385	3.307	0.758	118.4	125.2	0.085	0.175	0.885	0.314	0.046	0.944
	0.010	**0.000**									
3	0.053	3.238	0.751	121.2	127.9	0.086	0.180	0.885	0.313	0.046	0.942
	0.669	**0.000**									
12	0.091	3.292	0.759	114.6	121.3	0.082	0.173	0.890	0.315	0.043	0.945
	0.488	**0.000**									
	Exch. Rate - Change										
1	0.021	3.384	0.742	68.7	74.8	0.069	0.149	0.896	0.317	0.036	0.948
	0.717	**0.000**									
3	0.141	3.471	0.753	66.3	72.4	0.067	0.144	0.896	0.317	0.036	0.950
	0.029	**0.000**									
12	0.008	3.373	0.744	68.5	74.6	0.070	0.152	0.895	0.316	0.037	0.948
	0.903	**0.000**									

Table2.3 *concluded*

Note: 1. Wald Test p -value below $\hat{\Theta}$. 2. **Bold** entries show significance at 5% or below. 3. Ranges of different statistics: R^2 = [0,1], LPS = [0,∞), QPS = [0,2], KS = [0,1], Pietra Index (PI) = [-0.354,0.354], ER (Error Rate) = [0,1] and AUC (Area Under ROC Curve) = [0.5,1). 4. KS, AUC and PI have positive orientation meaning that a higher value implies a better fit, while QPS, LPS and ER are negatively oriented. 5. Estimation period: 1957:M1-2011M12 *except* M1 & M2 (1959:M1-2011:M12) and NEER (1975:M1-2011:M12).

Given the disappointing results for the static probit model, we introduce dynamics in the BCM model (2.18), see Tables 2.3, 2.4 and 2.5. The dynamic specifications in Tables 2.3 and 2.4 only contain S_{t-1} and π_{t-1} , respectively, because higher lag orders did not produce significantly improved fits or likelihoods. Let us first have a look at the in-sample fits in Table 2.3. The most striking observation is the dramatic rise in goodness of fit to approximately 76%. Given the strong persistence in US stock market bulls and bears, this is perhaps not too surprising. The two information criteria also point towards a significantly improved fit as compared to the static probit regression. As concerns its individual significance, the lagged bear market indicator turns out to be highly significant at all horizons. The downside of including the lagged dependent variable is that it seems to take over the explanatory power of the macro-financial indicators almost entirely: no indicator is found to be consistently significant across different time horizons.

As concerns the out-of-sample results, the diagnostics are approximately the same. This may be due to the fact that the lagged dependent variable as an explanatory element possibly overpowers the other exogenous variables.

Table 2.4: PROBIT: IN- AND OUT-OF-SAMPLE: MODEL $P_{t-k}(S_t = 1) = \Phi(\alpha + \beta x_{t-k} + \gamma \pi_{t-1})$

	$\hat{\Theta}$		**In-sample**			**Out-of-sample**					
k	x_{t-k}	π_{t-1}	R^2	**AIC**	**BIC**	**QPS**	**LPS**	**KS**	**PI**	**ER**	**AUC**
	TERM SPREAD - 10Y										
1	0.020	0.980	0.127	347.2	353.9	0.348	0.522	0.149	0.151	0.253	0.726
	0.000	**0.000**									
3	0.021	0.979	0.128	346.2	352.9	0.349	0.522	0.131	0.157	0.259	0.731
	0.000	**0.000**									
12	0.231	-0.104	0.036	369.2	375.9	0.379	0.565	0.034	0.046	0.258	0.600
	0.109	0.877									
	TERM SPREAD - 5Y										
1	0.026	0.984	0.140	342.8	349.5	0.345	0.515	0.165	0.147	0.244	0.735
	0.000	**0.000**									
3	0.027	0.983	0.141	341.9	348.6	0.346	0.516	0.145	0.145	0.251	0.736
	0.000	**0.000**									
12	0.344	-0.400	0.033	370.2	376.9	0.379	0.567	0.029	0.054	0.258	0.605
	0.216	0.734									
	INFLATION										
1	0.526	0.322	0.019	382.8	389.5	0.390	0.577	0.012	0.062	0.274	0.588
	0.027	0.251									
3	0.432	0.297	0.012	384.5	391.2	0.393	0.581	-0.002	0.043	0.274	0.576
	0.258	0.660									
12	0.058	0.932	0.015	375.9	382.6	0.388	0.576	0.000	0.041	0.252	0.565
	0.004	**0.000**									
	IND. PROD. - GROWTH										
1	-0.164	0.095	0.013	384.8	391.5	0.391	0.580	0.009	0.064	0.272	0.588
	0.026	0.749									
3	-0.042	-0.399	0.001	388.2	394.9	0.398	0.587	0	0.006	0.274	0.505
	0.527	0.716									
12	0.076	0.734	0.011	377.0	383.7	0.390	0.578	0.000	0.050	0.261	0.561
	NAR. MONEY - GROWTH										
1	0.045	0.853	0.006	374.7	381.4	0.398	0.587	0.000	0.032	0.274	0.529
	0.072	**0.000**									
3	0.047	0.843	0.005	374.2	380.9	0.399	0.588	0.000	0.021	0.277	0.526
	0.072	**0.000**									
12	0.145	0.141	0.002	366.4	373.1	0.393	0.582	0.004	0.026	0.271	0.554
	0.077	0.695									

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Table2.4 *Contd. from pre-page*

	Brd. Money - Growth										
1	0.105	0.945	0.057	358.2	364.9	0.377	0.560	0.056	0.077	0.265	0.646
	0.000	**0.000**									
3	0.129	0.934	0.064	355.5	362.1	0.375	0.558	0.103	0.074	0.253	0.650
	0.000	**0.000**									
12	0.270	0.830	0.064	347.1	353.8	0.368	0.551	0.105	0.104	0.254	0.648
	0.002	**0.000**									
	Unemp. Rate - Change										
1	0.403	0.175	0.004	387.9	394.6	0.396	0.585	0.000	0.032	0.274	0.537
	0.134	0.447									
3	0.092	-0.312	0.000	388.4	395.1	0.398	0.587	0.000	0.000	0.274	0.500
	0.739	0.510									
12	-0.262	0.467	0.002	379.9	386.6	0.394	0.583	0.000	0.009	0.270	0.517
	0.240	0.065									
	Fed. F Rate - Change										
1	0.138	0.988	0.078	363.2	370.0	0.361	0.547	0.194	0.093	0.233	0.654
	0.000	**0.000**									
3	0.128	0.988	0.068	365.9	372.6	0.367	0.553	0.123	0.090	0.248	0.645
	0.000	**0.000**									
12	0.086	0.804	0.005	378.9	385.6	0.391	0.581	0	0.034	0.256	0.519
	0.347	0.023									
	Exch. Rate - Change										
1	0.037	0.969	0.072	222.1	228.3	0.323	0.496	-0.032	0.123	0.224	0.687
	0.000	**0.000**									
3	0.039	0.962	0.067	222.8	228.9	0.325	0.499	-0.023	0.123	0.225	0.689
	0.000	**0.000**									
12	0.040	0.935	0.044	225.7	231.8	0.339	0.517	-0.006	0.091	0.229	0.653
	0.001	**0.000**									

Table2.4 *concluded*

Note: 1. Wald Test p -value below $\hat{\Theta}$. 2. **Bold** entries show significance at 5% or below. 3. Ranges of different statistics: R^2 = [0,1], LPS = [0,∞), QPS = [0,2], KS = [0,1], Pietra Index (PI) = [-0.354,0.354], ER (Error Rate) = [0,1] and AUC (Area Under ROC Curve) = [0.5,1). 4. KS, AUC and PI have positive orientation meaning that a higher value implies a better fit, while QPS, LPS and ER are negatively oriented. 5. Estimation period: 1957:M1-2011M12 *except* M1 & M2 (1959:M1-2011:M12) and NEER (1975:M1-2011:M12).

Encouraged by the statistical evidence for persistence of market conditions, we now test whether the perceived past beliefs (probability) of market fluctuations, π_t, have any impact. We argue that π_t implies the sense of severity of the regime. Therefore, inclusion of a lagged autoregressive term may add value to the forecast ability. Table (2.4) reports the results for the dynamic probit model specification where S_{t-1} is replaced by the autoregressive term, π_{t-1}. Interestingly, all the macro-variables now exhibit forecast ability for different horizons - i.e. some appear to be useful for short horizons while others exhibit predictive power for medium and long horizons. The results support the assertion that it is the perception and anticipation about the market conditions that move the price levels induced by, inter alia, the changes in macro-economic conditions as implied by various macro-financial variables. Broad money growth turns out to be significant in this specification as well, but now for all horizons. This possibly points to a market response to monetary policy.

Table 2.5: PROBIT: IN- AND OUT-OF-SAMPLE: MODEL $P_{t-k}(S_t = 1) = \Phi(\alpha + \beta x_{t-k} + \xi x_{t-1} \times S_{t-1}$

	$\hat{\Theta}$		In-sample			Out-of-sample					
k	x_{t-k}	$x_{t-1}.\S_{t-1}$	R^2	**AIC**	**BIC**	**QPS**	**LPS**	**KS**	**PI**	**ER**	**AUC**
	TERM SPREAD - 10Y										
1	0.781	-1.274	0.426	243.8	250.5	0.221	0.365	0.527	0.244	0.135	0.875
	0.000	**0.000**									
3	0.584	-1.049	0.374	261.9	268.6	0.232	0.394	0.530	0.232	0.140	0.873
	0.000	**0.000**									
12	0.293	-0.750	0.276	289.6	296.3	0.257	0.442	0.453	0.203	0.142	0.804
	0.000	**0.000**									
	TERM SPREAD - 5Y										
1	0.966	-1.609	0.444	237.1	243.8	0.211	0.355	0.603	0.248	0.123	0.875
	0.000	**0.000**									
3	0.685	-1.290	0.383	258.7	265.5	0.224	0.389	0.575	0.240	0.129	0.874
	0.000	**0.000**									
12	0.325	-0.956	0.291	284.4	291.1	0.243	0.434	0.472	0.237	0.117	0.817
	0.000	**0.000**									
	INFLATION										
1	-2.806	7.203	0.538	203.0	209.7	0.156	0.304	0.726	0.275	0.079	0.894
	0.001	**0.000**									
3	-1.041	6.094	0.485	222.1	228.8	0.174	0.334	0.638	0.276	0.076	0.898
	0.006	**0.000**									
12	-0.504	5.755	0.474	220.8	227.5	0.173	0.337	0.659	0.276	0.073	0.892
	0.111	**0.000**									
	IND. PROD. - GROWTH										
1	-0.347	0.381	0.029	379.5	386.3	0.391	0.572	-0.020	0.138	0.274	0.616
	0.000	**0.006**									
3	-0.041	0.042	0.001	388.1	394.9	0.398	0.587	0	0.011	0.273	0.500
	0.543	0.730									
12	0.090	0.071	0.005	379.2	385.9	0.392	0.581	0.000	0.044	0.266	0.540
	0.148	0.585									
	NAR. MONEY - GROWTH										
1	-1.028	2.263	0.245	297.2	303.8	0.282	0.464	0.356	0.195	0.174	0.816
	0.000	**0.000**									
3	0.020	1.375	0.153	327.0	333.7	0.295	0.513	0.313	0.242	0.098	0.777
	0.828	0.005									
12	0.165	1.460	0.173	312.5	319.2	0.280	0.497	0.337	0.236	0.104	0.813
	0.134	**0.007**									
	BRD. MONEY - GROWTH										
1	-1.458	5.265	0.623	166.2	172.92	0.132	0.257	0.745	0.293	0.065	0.927
	0.000	**0.000**									
3	-0.280	4.984	0.579	182.3	188.9	0.145	0.284	0.692	0.299	0.055	0.914
	0.175	**0.000**									
12	0.252	4.903	0.590	172.8	179.5	0.139	0.273	0.698	0.301	0.055	0.923
	0.209	**0.000**									
	UNEMP. RATE - CHANGE										
1	0.073	0.822	0.007	386.7	393.5	0.392	0.583	0.000	0.081	0.216	0.480
	0.356	0.161									
3	-0.069	0.918	0.007	386.1	392.8	0.392	0.584	0.000	0.112	0.192	0.524

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Table2.5 *Contd. from pre-page*

	0.969	0.265									
12	-0.269	0.770	0.006	378.6	385.3	0.390	0.581	0.000	0.086	0.219	0.540
	0.282	0.194									
	FED. F RATE - CHANGE										
1	0.138	-0.010	0.003	388.2	394.9	0.396	0.585	0.000	0.019	0.271	0.513
	0.389	0.939									
3	0.190	0.133	0.007	386.2	392.9	0.394	0.584	0.006	0.051	0.259	0.529
	0.108	0.482									
12	0.150	0.201	0.005	379.0	385.7	0.391	0.581	0.006	0.048	0.244	0.501
	0.265	0.301									
	ECCH. RATE - CHANGE										
1	0.051	0.021	0.006	236.7	242.8	0.345	0.529	0.000	0.053	0.219	0.551
	0.060	0.818									
3	0.078	0.065	0.014	234.5	240.6	0.343	0.526	0.000	0.043	0.218	0.572
	0.024	0.452									
12	0.040	0.070	0.006	233.89	239.9	0.350	0.536	0	0.048	0.213	0.534
	0.217	0.415									

Table2.5 *concluded*

Note: 1. Wald Test p -value below $\hat{\Theta}$. 2. **Bold** entries show significance at 5% or below. 3. Ranges of different statistics: R^2 = [0,1], LPS = [0,∞), QPS = [0,2], KS = [0,1], Pietra Index (PI) = [-0.354,0.354], ER (Error Rate) = [0,1] and AUC (Area Under ROC Curve) = [0.5,1). 4. KS, AUC and PI have positive orientation meaning that a higher value implies a better fit, while QPS, LPS and ER are negatively oriented. 5. Estimation period: 1957:M1-2011M12 *except* M1 & M2 (1959:M1-2011:M12) and NEER (1975:M1-2011:M12).

Finally, we added the lagged interaction term of the explanatory and the dependent variables $x_{t-p}.S_{t-q}$. We only consider the first lag of the interaction term, i.e., $p = q = 1$. Results are summarized in table 2.5. Both spread variables show predictability for all k's, while inflation, growth rates in industrial production and money supply again turn out to be significant for either short or long horizons. The fact that the interaction term remains significant despite the inclusion of macro-financial variables points to the regime persistence and the state contingency of the different economic variables.

2.5.4 Markov-switching regression model

In this sub-section we discuss the in-sample fit and out-of-sample forecasting power of a Markov-switching (MS) model that is an alternative to the dynamic BCM model class. The slope of the macro-financial indicator β is made regime-dependent. The bear state probabilities for the S&P 500 returns are conditioned on an exogenous macro-financial variable (see model (2.12)). Results are reported in Table 2.6. The table's left panel contains the coefficient estimates $\widehat{\theta}$ whereas the middle and right panels report in-sample and out-of-sample regression diagnostics. The bull and bear probability estimates for p_{00} and p_{11} are very high which is in line with the earlier observed strong persistence in bull and bear phases. The regime-dependence of the slope β is rather puzzling.

The slopes typically drop during bear phases and even change signs across bull and bear regimes. None of the variables is able to predict both states.

Table 2.6: MARKOV-SWITCHING MODEL: IN- AND OUT-OF-SAMPLE RESULTS. ($y_t = \alpha_{s_t} + \beta_{s_t} x_t + \sigma_{s_t} \varepsilon_t$)

	$\hat{\Theta}$				In-sample			Out-of-sample					
	p_{00}	p_{11}	$\hat{\beta}_0$	$\hat{\beta}_1$	LLIK	AIC	BIC	QPS	LPS	KS	PI	ER	AUC
Spread - 10Y	0.948	0.976	0.146	-0.344	-1859	3737	3782	0.793	1.451	0.181	0.064	0.273	0.631
	0.000	**0.000**	0.407	0.085									
Spread - 5Y	0.932	0.807	-0.060	-0.638	-1857	3735	3780	0.399	0.611	0.319	0.113	0.253	0.702
	0.000	**0.000**	0.741	0.363									
Inflation	0.945	0.830	-0.819	-0.855	-1857	3735	3780	0.412	0.629	0.294	0.104	0.253	0.689
	0.000	**0.000**	0.215	0.640									
Ind. Prod. - Grth.	0.906	0.844	0.213	0.014	-1881	3782	3827	0.335	0.517	0.491	0.174	0.229	0.814
	0.000	**0.000**	0.586	0.975									
Nar. Money - Grth.	0.992	0.997	0.294	-0.271	-1803	3627	3670	1.041	2.443	0.130	0.046	0.276	0.540
	0.000	**0.000**	0.419	0.360									
Brd. Money - Grth.	0.944	0.832	0.397	-0.965	-1796	3611	3656	0.415	0.632	0.292	0.103	0.255	0.685
	0.000	**0.000**	0.431	0.578									
Unemp. - Change	0.927	0.798	2.780	-4.536	-1855	3730	3775	0.360	0.560	0.387	0.137	0.247	0.749
	0.000	**0.000**	**0.005**	0.130									
Fed F Rate - Change	0.923	0.868	-3.578	-0.071	-1846	3711	3756	0.386	0.671	0.367	0.130	0.237	0.742
	0.000	**0.000**	**0.000**	0.893									
Exch. Rate - Change	0.957	0.721	-0.227	-0.643	-1261	2542	2583	0.342	0.555	0.382	0.135	0.205	0.733
	0.000	**0.000**	**0.043**	0.221									

Notes: 1. p-values below $\hat{\Theta}$. 2. **Bold** entries show significance at 5% or below. 3. Ranges of different statistics: LPS = [0,∞), QPS = [0,2], KS = [0,1], Pietra Index (PI) = [-0.354,0.354], Error Rate (ER) = [0,1] and Area Under ROC Curve AUC = [0.5,1). 4. KS, AUC and PI have positive orientation meaning that a higher value implies a better fit, while QPS, LPS and ER are negatively oriented. 5. Estimation period: 1957:M1-2011M12 *except* M1 & M2 (1959:M1-2011:M12) and NEER (1975:M1-2011:M12).

For the out-of-sample evaluation, we employ the same criteria as for the binary choice models. These statistics are reported in last six columns of table 2.6. The best out-of-sample performance is obtained for growth in industrial production as QPS, LPS and ER have comparatively lower values for this variable and comparatively higher values for PI and AUC. Summarizing, the MS model's out-of-sample results are not encouraging when compared with the binary choice models. However, we need a non-nested hypothesis test like the earlier discussed Diebold-Mariano procedure to formally compare the forecasting power of MS models with the BCM class. This will be next section's focus.

2.5.5 Comparison of forecasts

In this section, we compare the inter- as well as intra-model forecasting ability of different models. We report results for the Clark and West (2007) (nested models) and the Diebold and Mariano (1995) (non-nested models) in tables 2.7 and 2.8, respectively. The Clarke and West p-values in Table 2.7 compare the forecast ability of the simple static model (2.18) with various nested dynamic specifications. By definition, the Clarke and West test is only suited to compare models' forecasting power within the probit (panel A) or logit class (panel B). In line with the out-of-sample results, dynamic probit (logit) specifications generally outperform static probit (logit) specifications.

As concerns non-nested model comparisons, we compare probit models with logit models (left panel in Table 2.8), probit models with MS models (middle panel in Table 2.8) and logit models with MS models (right panel in Table 2.8). The left panel shows that probit analysis only outperforms logit analysis when using the interactive dynamic model (XZ); but in all other cases, the distributional assumption for $F(\cdot)$ in (2.6) seems irrelevant.

The remaining forecast comparisons in the middle and right panel of Table 2.8 convincingly show that dynamic binary response models outperform Markov-switching regressions. This confirms earlier research by Birchenhall et al. (1999) and Candelon, Dumitrescu and Hurlin (2012) who already established that logit model performs better than Markov-switching specification.

Overall, the macro-financial variables offer a useful set of variables to forecast the upturns and the downturns on the stock market.

Table 2.7: Clark-West (2007) Test for equal MSPE - Probit & Logit: Static (**X**) versus Dynamic Specifications

	Dynamic Specifications								
k	**XY**	**XZ**	**XP**	**XY**	**XZ**	**XP**	**XY**	**XZ**	**XP**
Panel A: Probit									
	Term Spread - 10Y			Term Spread - 5Y			Inflation		
1	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.253
3	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.414
12	**0.000**	**0.000**	0.224	**0.000**	**0.000**	0.255	**0.000**	**0.000**	0.014
	Ind. Prod. - Growth			Nar. Money - Growth			Nar. Money - Growth		
1	**0.000**	0.104	0.511	**0.000**	**0.000**	0.102	**0.000**	**0.000**	**0.000**
3	**0.000**	0.321	0.417	**0.000**	**0.000**	0.055	**0.000**	**0.000**	**0.000**
12	**0.000**	0.195	**0.010**	**0.000**	**0.000**	0.386	**0.000**	**0.000**	**0.000**
	Unemp. Rate - Change			Fed Funds Rate - Chg.			NEER Index - Change		
1	**0.000**	**0.004**	0.390	**0.000**	0.819	**0.000**	**0.000**	0.051	**0.000**
3	**0.000**	**0.011**	0.386	**0.000**	0.163	**0.000**	**0.000**	0.101	**0.000**
12	**0.000**	**0.029**	0.221	**0.000**	0.116	0.105	**0.000**	0.111	**0.000**
Panel B: Logit									
	Term Spread - 10Y			Term Spread - 5Y			Inflation		
1	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.029**	**0.000**	**0.000**	0.272
3	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.440
12	**0.000**	**0.000**	0.229	**0.000**	**0.000**	0.262	**0.000**	**0.000**	**0.014**
	Ind. Prod. - Growth			Nar. Money - Growth			Brd. Money - Growth		
1	**0.000**	0.162	0.542	**0.000**	**0.000**	0.119	**0.000**	**0.000**	**0.000**
3	**0.000**	0.294	0.414	**0.000**	**0.000**	0.059	**0.000**	**0.000**	**0.000**
12	**0.000**	0.152	**0.009**	**0.000**	**0.000**	0.409	**0.000**	**0.000**	**0.000**
	Unemp. - Change			Fed. F Rate - Change			Exch. Rate - Change		
1	**0.000**	**0.001**	0.390	**0.000**	0.115	**0.000**	**0.000**	**0.028**	**0.000**
3	**0.000**	**0.004**	0.386	**0.000**	0.129	**0.000**	**0.000**	0.060	**0.000**
12	**0.000**	**0.014**	0.221	**0.000**	0.105	0.119	**0.000**	0.067	**0.000**

Notes: 1. **X**: $\pi_t = \alpha + \beta x_{t-k}$. 2. **XY**: $\pi_t = \alpha + \beta x_{t-k} + \delta S_{t-1}$. 3. **XZ**: $\pi_t = \alpha + \beta x_{t-k} + \xi x_{t-1}.S_{t-1}$. 4. **XP**: $\pi_t = \alpha + \beta x_{t-k} + \gamma \pi_{t-1}$. 5. CW Test H_0 : Equal MSPE *v/s* H_1 : Dynamic specification performs better than static (**x**). 6. *p*-values with ***bold*** entries indicating significance at 5% or below.

2.6 Predicting Bear Markets and investors' market timing

In this section we illustrate that dynamic binary response models may be a useful toolkit for market timing and active portfolio rebalancing. We define three active trading strategies that allow for monthly rebalancing and compare their performance with a passive buy-and-hold strategy. Each strategy starts with an initial investment of 1US$ (December 1979) and ends in December 2011.[16] We recursively forecast one-month ahead bear probabilities using each of the four Probit models (X, XY, XZ and XP) in combination with each macro-financial variable. Next, we compare this bear probability with three cut-off points (τ=40%, 50% and 70%) to classify whether the market is in a bear or bull state. More specifically, if the forecast probability is greater than or equal to the threshold, we label the market to be in a bear state and go on to active

[16]The starting point is arbitrary and the ranking of trading strategies is robust to changing the starting point.

Table 2.8: Diebold-Mariano (1995) Test for equality of forecasts

	Probit vs Logit				Probit vs MSReg				Logit vs MSReg			
	X	XY	XZ	XP	X	XY	XZ	XP	X	XY	XZ	XP
Term Spread - 10Y												
1	0.071	0.909	**0.001**	0.426	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
3	0.362	0.524	**0.004**	0.487	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
12	0.443	0.741	**0.005**	0.445	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Term Spread - 5Y												
1	0.151	0.693	**0.000**	**0.000**	0.200	**0.000**	**0.000**	**0.000**	0.196	**0.000**	**0.000**	0.602
3	0.460	0.508	**0.000**	0.400	**0.009**	**0.000**	**0.000**	**0.000**	**0.009**	**0.000**	**0.000**	**0.000**
12	0.356	0.981	**0.000**	0.388	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Inflation												
1	0.966	0.972	**0.000**	0.519	**0.027**	**0.000**	**0.000**	**0.025**	**0.027**	**0.000**	**0.000**	**0.025**
3	0.720	0.807	**0.000**	0.854	**0.007**	**0.000**	**0.000**	**0.007**	**0.007**	**0.000**	**0.000**	**0.007**
12	0.710	0.980	**0.001**	0.583	**0.002**	**0.000**	**0.000**	**0.002**	**0.002**	**0.000**	**0.000**	**0.002**
Ind. Prod. - Growth												
1	0.333	0.812	0.352	0.340	0.298	**0.000**	0.300	0.296	0.308	**0.000**	0.289	0.305
3	0.799	0.490	0.625	0.760	**0.011**	**0.000**	**0.011**	**0.011**	**0.011**	**0.000**	**0.011**	**0.011**
12	0.955	0.650	0.455	0.530	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Nar. Money - Growth												
1	0.460	0.559	**0.000**	0.598	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
3	0.968	0.394	**0.000**	0.753	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
12	0.593	0.591	**0.000**	0.911	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Brd. Money - Growth												
1	0.460	0.658	**0.000**	0.824	0.054	**0.000**	**0.000**	**0.004**	0.054	**0.000**	**0.000**	**0.005**
3	0.860	0.531	**0.003**	0.888	**0.012**	**0.000**	**0.000**	**0.003**	**0.012**	**0.000**	**0.000**	**0.003**
12	0.707	0.585	**0.042**	0.917	**0.001**	**0.000**	**0.000**	**0.000**	**0.001**	**0.000**	**0.000**	**0.000**
Unemp. Rate - Change												
1	0.807	0.559	**0.010**	0.806	0.255	**0.000**	0.407	0.255	0.255	**0.000**	0.519	0.255
3	0.930	0.849	**0.055**	1.000	0.121	**0.000**	0.063	0.121	0.121	**0.000**	**0.049**	0.121
12	0.814	0.735	0.218	0.898	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Fed. Fund Rate - Change												
1	0.350	0.275	0.234	**0.000**	0.531	**0.000**	0.533	0.016	0.526	**0.000**	0.517	0.784
3	0.393	0.773	0.281	0.450	**0.017**	**0.000**	**0.016**	**0.001**	**0.016**	**0.000**	**0.016**	**0.001**
12	0.392	0.614	0.342	0.419	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Exch. Rate - Change												
1	0.589	0.865	0.076	0.172	0.468	**0.000**	0.433	0.063	0.467	**0.000**	0.365	0.069
3	0.504	0.139	0.202	0.513	0.358	**0.000**	0.294	0.165	0.359	**0.000**	0.261	0.175
12	0.714	0.965	0.247	0.353	0.083	**0.000**	0.073	**0.045**	0.083	**0.000**	0.068	**0.046**

Notes: 1. **X**: $\pi_t = \alpha + \beta x_{t-k}$. 2. **XY**: $\pi_t = \alpha + \beta x_{t-k} + \delta S_{t-1}$. 3. **XZ**: $\pi_t = \alpha + \beta x_{t-k} + \xi x_{t-1}.S_{t-1}$. 4. **XP**: $\pi_t = \alpha + \beta x_{t-k} + \gamma \pi_{t-1}$. 5. DM test H_0 : equal predictive accuracy of model-I and model-II v/s H_1 : model-I performs better than model-II. 6. p-values reported with **bold** entries significant at 5% or below.

re-balancing.

A first investment strategy boils down to either investing in a 1-year treasury bond if the BCM forecast generates one-month ahead bear conditions or investing in the S&P500 index otherwise, see e.g. Pesaran and Timmermann (1995) or Chen (2009) for earlier applications. A second strategy consists of either going short when the stock market is predicted to enter a bear state or staying long otherwise. This long/short strategy is quite popular amongst hedge funds: Fung and Hsieh (2011) argues that around 40% of hedge fund managers follow this strategy. A third strategy consists of buying when the model forecasts a trough and selling when the market is at a peak. This means that investors are in the market in normal or bull conditions but are out of market in turbulent times, see (Dai et al. (2010)).[17]

One of the problems with existing research on active trading strategy profitability is that they do not take sufficient account of the impact transaction costs can have on the trading strategy's profitability. To address this concern, we set transaction costs equal to 50 and 10 basis points per trade in stocks and bonds, respectively, see Pesaran and Timmermann (1995); Balduzzi and Lynch (1999); Han et al. (2011); Pollin and Heintz (2011). As concerns the long/short strategy, the costs are set at 100 basis points for going short as this is often very expensive, see Diether et al. (2009)[18].

The results of the exercises are reported in table 2.9. Panel A shows the passive strategy's monthly compound return as well as the return from a similar strategy when investing in a 1-year treasury security. The results from the active trading strategies are included in panel B of the table.

[17]This last strategy assumes that the investor has a current account with some discount broker who acts only when advised by the investor. Furthermore, the proceeds of a sale remain with the broker and earn zero interest. Of course the idea is that zero return is preferable over the negative return during the downturns. This assumption then makes this active strategy comparable with the passive buy-and-hold one.

[18]Our long/short strategy is not market-neutral. We also assume that (i) the proceeds from the short-sell remain with the broker, (ii) the proceeds do not earn any interest from the broker, and (iii) the margin is settled when the short position is closed. As reported by Boehmer et al. (2008), short-selling is mostly employed by institutional investors (72% versus individuals 2%). Thus, the assumptions are likely to hold.

Table 2.9: MONTHLY COMPOUND RETURN (%) FROM BUY-AND-HOLD AND ACTIVE RE-BALANCING STRATEGIES

PANEL A: BUY-AND-HOLD STRATEGY

Compound monthly benchmark return from S&P 500 index : 0.6376

Compound monthly return from 1-year US Treasury bill : 0.4536

PANEL B: ACTIVE RE-BALANCING STRATEGIES - *Transaction costs: (i) Long position - Stocks 50bps and Bonds 10bps (ii) Short position - 100 bps*

MODEL	X			XY			XZ			XP		
τ	40%	50%	70%	30%	50%	70%	40%	50%	70%	40%	50%	70%
STRATEGY: STOCKS OR 1-YEAR TREASURY BILLS												
Spread - 10Y	1.933	1.349	0.639	4.327	4.327	4.327	4.751	3.959	3.215	2.067	1.349	0.813
Spread - 5Y	1.693	1.121	0.639	4.327	4.327	4.327	4.728	3.943	3.227	1.692	1.121	0.639
Inflation	1.430	0.639	0.639	4.327	4.327	4.327	4.189	3.710	2.693	1.363	0.639	0.639
Ind. Prod. Grth.	1.668	0.704	0.639	4.327	4.327	4.327	2.576	0.798	0.639	1.574	0.704	0.639
Nar. Money - Grth	2.190	0.676	0.639	4.327	4.327	4.327	5.171	4.415	2.943	2.623	0.756	0.639
Brd. Money - Grth	4.005	1.556	0.639	4.327	4.327	4.327	4.335	3.943	3.770	4.286	1.775	0.639
Unemp. - Chng.	0.688	0.639	0.639	4.327	4.327	4.327	1.074	0.639	0.639	2.538	1.306	0.639
FFR - Chng.	1.561	0.916	0.822	4.327	4.327	4.273	1.493	0.890	0.768	2.230	1.756	1.315
Exch. Rate - Chng.	1.708	0.788	0.639	4.327	4.327	4.327	1.714	0.932	0.700	2.976	1.885	1.098
STRATEGY: LONG-SHORT												
Spread - 10Y	0.689	0.632	0.640	1.388	1.388	1.388	1.215	1.162	1.089	0.651	0.632	0.632
Spread - 5Y	0.607	0.554	0.640	1.388	1.388	1.388	1.148	1.122	1.054	0.698	0.554	0.640
Inflation	0.592	0.640	0.640	1.388	1.388	1.388	1.286	1.174	0.925	0.664	0.640	0.640
Ind. Prod. Grth.	0.638	0.724	0.640	1.388	1.388	1.388	0.286	0.561	0.640	0.680	0.724	0.640
Nar. Money - Grth	0.132	0.610	0.640	1.388	1.388	1.388	0.561	0.580	0.955	0.066	0.583	0.640
Brd. Money - Grth	0.013	0.324	0.640	1.388	1.388	1.388	1.080	1.289	1.336	0.004	0.262	0.640
Unemp. - Chng.	0.625	0.640	0.640	1.388	1.388	1.388	0.652	0.640	0.640	0.303	0.584	0.640
FFR - Chng.	0.525	0.596	0.649	1.388	1.388	1.401	0.829	0.660	0.659	0.557	0.644	0.681
Exch. Rate - Chng.	0.492	0.613	0.640	1.388	1.388	1.388	0.611	0.649	0.647	0.364	0.374	0.536
STRATEGY: BUY-LOW, SELL-HIGH												
Spread - 10Y	0.709	0.651	0.626	0.924	0.924	0.924	0.952	0.860	0.846	0.630	0.651	0.570
Spread - 5Y	0.689	0.605	0.626	0.924	0.924	0.924	0.896	0.881	0.784	0.706	0.605	0.626
Inflation	0.591	0.626	0.626	0.924	0.924	0.924	0.848	0.802	0.778	0.638	0.626	0.626
Ind. Prod. Grth.	0.609	0.668	0.626	0.924	0.924	0.924	0.339	0.577	0.626	0.647	0.668	0.626
Nar. Money - Grth	0.277	0.615	0.626	0.924	0.924	0.924	0.617	0.553	0.699	0.335	0.596	0.626
Brd. Money - Grth	0.192	0.412	0.626	0.924	0.924	0.924	0.789	0.905	0.918	0.202	0.451	0.626
Unemp. - Chng.	0.607	0.626	0.626	0.924	0.924	0.924	0.590	0.626	0.626	0.421	0.588	0.626
FFR - Chng.	0.530	0.567	0.614	0.924	0.924	0.931	0.724	0.592	0.624	0.587	0.667	0.688
Exch. Rate - Chng.	0.574	0.562	0.626	0.924	0.924	0.924	0.581	0.580	0.595	0.548	0.473	0.570

Notes: (i) The threshold, τ, represents the cut-off probability at or above which the market is assumed to be in bear state. *X, XY, XZ, XP* represent, respectively, the static, dynamic, interactive-dynamic and autoregressive probit models (see note to table 2.8). (iii) Based on recursive forecasts from 1980M1-2011M12

First and foremost, upon comparing the strategies in terms of return generation, the first strategy performs best and the third strategy performs worst while the long-short strategy takes on an intermediate position. Under the other two strategies, the returns from the long/short strategy could have been marred by both higher transaction costs as well as by possible forecast inaccuracies, both of which weigh heavily on the overall return. The third strategy's return is obviously eroded by intermediate bear market episodes that do not offer return. However, even in the latter cases the performance of the strategies is generally better than the passive strategy. Active strategies have also been found to accrue positive (abnormal) returns by, e.g., Cohen et al. (2007) and Diether et al. (2009) (for short-selling) and Pesaran and Timmermann (1995); Marquering and Verbeek (2004) (for other related strategies). A second observation from the table is that dynamic strategies based on model XY outperform other dynamic strategies in the sense that XY$\succ$XZ$\succ$XP. Third, it is also interesting to compare the returns based on predictions conditioned on different macro-financial variables. The forecasts based on term spreads, inflation and money supply growth typically yield better results than for other variables.

Summarizing, active portfolio re-balancing based on forecasts based on dynamic probit models yields higher returns as compared to both a static probit model as well as a passive strategy. More specifically, forecasts based on a threshold of 40% in combination with dynamic BCM specifications that include term spreads, inflation or money supply growth produce optimal results.

2.7 Conclusion

In this chapter we compare different approaches towards predicting the bear conditions on the US stock market. We extract the bulls and bears episodes using both parametric (Markov-switching) as well as non-parametric (Bry and Boschan (1971)) techniques. Next, we compare the forecasting ability of linear predictive regressions, static and dynamic binary choice models as well as Markov-switching model. All these models make use of the same regressors set of (lagged) macro-financial variables.

We test whether the forecasting performances statistically differ across model specifications using the Clark and West (2007) test (nested model comparison) and the Diebold and Mariano (1995) test (non-nested model comparison). Our results show that dynamic extensions of simple probit and logit models both increase in- and out-of-sample fits compared to their static counterparts. Also, the dynamic probit model has a superior forecasting power as compared to a dynamic logit specification when one includes interaction terms between the cycle dummy and the macro/financial variable. Non-nested hypothesis testing using the Diebold and Mariano (1995) shows that binary choice models generally outperform the Markov-switching model in terms of forecasting power regardless the inclusion or structure of the dynamics included in the BCM model class. As concerns the statistical significance and out of sample forecasting ability of the macro-financial variables, the 10- and 5-year term spreads, inflation and changes in industrial production, money supply and funds rate turn out to be significant at different forecast horizons in static as well as various dynamic specifications of binary response models. We also argue that dynamic binary choice models can be a useful tool for investors and policymakers. We empirically illustrate that active portfolio investors using dynamic models for the BCM class to generate buy and sell signals outperform active investors using more rudimentary static models as well as passive investors. Finally, we believe that dynamic binary choice models may constitute a valuable addition to the toolkit of financial regulators and supervisors. These novel techniques may help them to improve upon existing Early Warning Systems to foresee financial crises and to monitor financial instability.

Chapter 3

Predicting Exchange Rate Cycles Utilizing Risk factors

3.1 Introduction

The exchange rate, the relative price of one currency in terms of another, is a primary macroeconomic indicator of the international competitiveness of a country. Attempts have been made to reconcile movements in the exchange rate with the movements of macroeconomic variables. The most familiar attempts have been the monetary models suggested by Mussa (1976) and Frenkel (1976) and the portfolio balance models proposed by Kouri (1976) and Dornbusch et al. (1980). However, Meese and Rogoff (1983) found that the forecasting power of these models was outperformed by a simple random walk model. Subsequently, Mark (1995) and Chinn and Meese (1995) reported long term predictability employing fundamentals, but Berkowitz and Giorgianni (2001) and Faust et al. (2003) cast doubts on these results as well. A more recent set of exchange rate determination models includes the so-called Taylor rule models proposed by Engel and West (2005, 2006) for which subsequent empirical studies, e.g., Molodtsova and Papell (2009) and Wang and Wu (2012), find evidence of short-term predictability. These inconsistent results about the predictability of models employing macroeconomic fundamentals led researchers to search for micro foundations. For example, Evans and Lyons (1999); Berger et al. (2008) utilized order flows and found that these flows perform better than macroeconomic fundamentals.

Credit is due to Mussa (1976) and Frenkel and Mussa (1985), who recognized the exchange rate as an asset price and maintained that ' the basic idea of the asset price approach to the exchange rate is that essentially the same theory of the determination of the prices of common shares is relevant to the determination of the exchange rate'. Mussa observed that if exchange rate is an asset price, it must be determined by the same basic forces that determine other asset prices (see also Obstfeld and Rogoff, 1996; Engel and West, 2005). With this perspective in mind, we note that periods of upswings (bulls) and downswings (bears) have been observed and studied in another major asset class, viz., equity prices. Likewise, cycles in the exchange rate have long been recognized by market practitioners who attempt to create profitable trading strategies by tracking these cyclical movements. Early examples of such studies are Poole (1967) and MacDonald and Young (1986). They formulate trading strategies whereby buy/sell orders are initiated on the basis of identified peaks/troughs and bulls/bears in the exchange rate. The determination of cyclical periods has, however, been subjective, and market practitioners mostly use less technically sophisticated analysis, e.g., a moving average. While the identification and prediction of bear episodes in the equity market using formal econometric methods is widespread (see, e.g., Edwards et al., 2003b; Pagan and Sossounov, 2003; Candelon et al., 2008b; Chen, 2009), comparable studies are conspicuously missing from the exchange rate literature. This omission exists despite the fact that, unlike equity prices, exchange rates do not exhibit trending behavior (a general upward or downward movement) but are essentially cyclical in nature. Whether these cycles move in consonance with real business cycles is ambiguous; however, investor expectations coupled with government intervention (or even commitment) to maintain a stable rate (within a band) induces cycles in the exchange rate. The Japanese government, for example, frequently intervenes in the foreign exchange market to maintain a weak, and hence competitive, yen; the Swiss National Bank supported the franc to reverse sustained appreciations post-floatation in the early 1980s and again in 2009 (Rich, 1990; SNB, 2009; Gerlach et al., 2011); the Reserve Bank of Australia frequently intervened in the exchange market after floating the dollar in 1983 but has recently announced the intention to intervene only during times of market dysfunction (Newman et al., 2011). Interventions to reduce the volatility of currencies are more frequent in emerging markets (see contributing papers in BIS, 2005). We thus try to fill this gap in the literature by extracting and explaining cycles in the exchange rate. Following the empirical studies on equity market bears, we employ a nonparametric algorithm Bry and Boschan (1971) to identify the turning points (peaks/troughs) in a country's exchange rate series, subject to censoring criteria. Based on the existing empirical literature examining currency crises, the censoring rules and thresholds that are most relevant to exchange rate behavior were determined. Consistent with the norms of the forex market, we then identify the periods from trough to peak as 'bear' and periods from peak to trough as 'bull'. Because these cycles are essentially binary, we use simple (static) and dynamic versions of binary choice models. The appropriateness of a dynamic model, which includes the lagged dependent variable on the right hand side, is suggested by the fact that cyclical phases tend to persist. The addition of a lagged dummy variable can be conceptualized as information augmentation. Further, discrete choice models help capture the hypothesized non-linearities in the exchange rate data (see e.g., Hsieh, 1989; Bilson, 1990). Incidentally, the binary choice framework has already been employed in the exchange rate literature for prediction of currency crises and identification of contagion (see e.g., Frankel and Rose, 1996; Berg and Pattillo, 1999; Candelon, Dumitrescu and Hurlin, 2012). By focusing on the bull and bear states, we attempt to capture the broad ups and downs in the currency market. Therefore, not only crises but also sustained periods of depreciation of a domestic currency vis-a-vis a foreign currency, which may arise due to the arrival of relevant news, can also be captured. Deviating from the mainstream literature on exchange rate determination, which includes both monetary and real variables in the models, we focus on deviations in leading parity conditions, viz., the uncovered interest rate parity (UIP), the relative purchasing power parity (RPPP), the equity market return differentials, the yield spread and the TED spread. These deviations and spreads represent broad risk factors in the foreign exchange market. These variables imply the risk/inflation premium, the liquidity/credit risk premium and the term premium. Given that we adopt an asset price perspective of the exchange rate and that these factors embody the expectations of the agents, the choice of variables seems plausible because expectations about future economic conditions move the asset markets. By employing these variables, we also aim to highlight commonly observed violations of the UIP and RPPP conditions. Moreover, the equity market return differentials that give rise to pseudo-parity conditions in which the commodity price index is replaced by the equity price index in PPP and the equity market returns are used instead of bond market returns in UIP. That substitution, if significant, may imply portfolio switching from the equity to the forex market.

The violation of UIP is of particular interest to currency traders. Interest rate parity predicts a depreciation of high interest currencies, which has been contradicted empirically. This violation gives rise to currency carry trade activities. The carry traders exploit international interest rate differentials and invest in high interest rate currencies by borrowing in a low interest rate currency. Over a sustained period, Australian and New Zealand dollars have been investment targets while the Swiss franc and the Japanese yen have been major funding currencies. While the volume of this activity is not precisely known, swings in major currencies have been found to be driven by the unwinding of carry trades (Cavallo, 1984; Galati et al., 2007; Christiansen et al., 2011). Indeed, between August 2008 and October 2008, in the wake of global turmoil due to the US sub-prime crisis, the unwinding of carry trades brought exceptional volatility to the forex market; a typical investment currency lost 2.6 years of yield advantage during that brief period of tumult (McCauley and McGuire, 2009). We noted that there have been frequent government interventions in the exchange rate market to reverse sustained appreciations of domestic currencies; future intervention by the central banks of target currencies would deal a devastating blow to carry traders and could produce upheaval in the market. Any indication of action by a central bank could help investors avoid probable losses. We believe that our simple framework, which exploits risk factors that represent market expectations, is a step in that direction.

Our empirical exercise considers the bilateral exchange rates of six advanced economies representing a mix of funding, target and reserve currencies. Our results indicate that violations of UIP are exploited by currency traders who earn abnormal returns by engaging in carry trades. When unspotted (static probit), deviations from UIP may persist for over a year. However, once spotted and the information is integrated by the market (dynamic probit), and violations become a short run phenomenon. Indeed, our dynamic model predicts a change of course in the exchange rate, e.g., from appreciation to depreciation over a particular forecast horizon. We consider these reversals to be an indication of the cyclical nature of exchange rates. In concrete terms, these reversals could cause the unwinding of carry trade positions; they could also produce liquidity pressures in the market and lead to the liquidity spirals envisioned in Brunnermeier and Pedersen (2009). In fact, an increase in the TED spread, which we employ to measure funding pressures in the market, also predicts an appreciation of funding currencies and a corresponding depreciation of investment currencies, implying an amplification of losses on carry trades. Interestingly, deviations from RPPP also appear to be short-term phenomena. Based on the comparison of results from our static and dynamic probit models, we believe that the short-term nature of violations of UIP and RPPP are due to agents revising the beliefs given new information. Consistent with the fact that the countries under study either implicitly or explicitly target inflation, any relative increase in inflation expectations leads to a depreciation of the domestic currency. Our dynamic probit model predicts depreciation from inflation expectations for upto a quarter and a subsequent reversal. This prediction is consistent with previous theoretical and empirical findings in the literature, e.g., overshooting. The result of a pseudo-parity condition where equity market return is substituted for inflation and bond return in RPPP and UIP points to possible portfolio switching between two equity markets in the short run. Finally, the relative term spread predicts depreciation cycles (bears) only in the static probit model but not in the dynamic specification. This finding contradicts the robust predictability of the relative term spread in previous business cycle studies.

The results of the study will be useful for both policy makers and market practitioners. The former can utilize this framework to obtain early warnings of possible sustained currency depreciation or appreciation. Such signals will be useful for policy interventions in the market to stabilize the value of the currency and, hence, maintain international competitiveness. Investors, which include sovereign wealth funds[1], can employ the framework to form trading strategies, hedge their positions as well as re-balance their carry trade positions.

The rest of the paper is organized as follows. In section 3.2, we briefly review the existing literature. Section 3.3 discusses the model, the determination of cyclical episodes and the variables used to explain the cyclical behavior of exchange rates. In section 3.4, we discuss our data and the empirical findings, and section 3.5 concludes.

[1]A voluntary group of 24 sovereign wealth funds (SWF) have formed an international forum under the guidance of the International Monetary Fund (IMF). In 2008, the group agreed to generally accepted principles and practices (GAAP), known as the 'Santiago Principles', which provide guidance for investment policy, risk management, etc. As a result of these principles and the losses suffered by funds in the wake of the recent financial crisis, SWFs are likely to focus on short-term investments.

3.2 Brief Literature Review

With a daily turnover of $4 trillion in April 2010, the foreign exchange (forex) market is the largest decentralized market by volume. This market is highly globalized; 65% of these transactions represented cross-border activity during the same period (see, King and Rime, 2010). Therefore, this financial asset class has received considerable attention from researchers for a long time. The research on the forex market has been multifaceted. One strand attempts to theoretically reconcile the exchange rate with macroeconomic fundamentals, especially in post-Bretton Woods era. Mussa (1976) and Frenkel (1976), for example, emphasized the monetary link by exploiting the observation that the exchange rate is the price of one currency in terms of the other and should be determined by the supply and demand for money in two countries. This observation led to monetary models of the exchange rates. The in-sample empirical performance of monetary models was satisfactory when data up to the late 1970s was included (see e.g., Bilson, 1978; Dornbusch, 1979; Frankel, 1979), but the performance of these models deteriorated when the timeline was extended to the 1980s (see, e.g., Haynes and Stone, 1981; Frankel, 1982; MacDonald and Taylor, 1989). The exchange rate has also been expressed in terms of the demand and supply of domestic and foreign financial assets, giving rise to the portfolio balance model (see e.g., Kouri, 1976; Dornbusch and Fischer, 1980). However, in an influential paper, Meese and Rogoff (1983) put monetary and portfolio balance models to an empirical analysis and conclude that out-of-sample, a random walk model performs as better as any of these models. This paper sparked a vast empirical literature on the exchange rate determination in which some researchers have found evidence of predictability over long time horizons (e.g., Mark, 1995; Chinn and Meese, 1995; Mark and Sul, 2001; Kilian and Taylor, 2003), while others cast doubts on either the results or the methods employed (see e.g., Berkowitz and Giorgianni, 2001; Rapach and Wohar, 2002; Faust et al., 2003). Another link between real exchange rates and monetary policy has been explored via the Taylor rule in Engel and West (2005, 2006). These models have been shown to outperform monetary models in forecasting changes in the exchange rate over short time horizons (see, e.g., Molodtsova and Papell, 2009; Wang and Wu, 2012).

Another strand of the literature focuses on the market micro-structure and recognizes that the market is largely decentralized; a comparatively large trading volume occurs mostly between market-makers. For example, Evans and

Lyons (1999); Berger et al. (2008) examine order flows (net buyer- and seller-initiated orders) and conclude that these variables have more explanatory power than macroeconomic variables. This line of research also tests the efficiency of the forex market by employing technical analysis and the chartists approach to form trading strategies, which outperform the buy-and-hold strategy. Considerable profits have been accrued by these strategies thus casting doubts on the efficiency of the market (see e.g., Poole, 1967; MacDonald and Young, 1986; Taylor and Allen, 1992).

Yet another strand of studies is influenced by the incidence of speculative attacks and currency crises. The so-called first generation models are based on the seminal paper by Krugman (1979). Krugman demonstrates that the creation of excessive domestic credit vis-a-vis growth in money demand causes depletion of foreign exchange reserves. This policy, which is *inconsistent* with pegged exchange rates, leads to speculative attacks on a domestic currency. However, the second generation models demonstrate that speculative attacks are possible even in presence of *consistent* policies if authorities face the temptation to devalue a currency. Speculators that know about this temptation will attack the currency and produce a currency crash in a self-fulfilling prophesy (see, e.g., Obstfeld, 1996). The third generation models, motivated mainly by the ASEAN currency crisis in 1997-98, incorporate the role of financial intermediaries in the generation of crises. These institutions may engage in activities that raise money in safer markets at lower interest rates and then lend in riskier markets at higher rates. Moral hazard can exist when the liabilities of these institutions are perceived to benefit from implicit government guarantees. Excessive lending in riskier markets may lead to asset price bubbles, which may burst when unsustainable and produce financial crises, including currency crises (see, e.g., McKinnon and Pill, 1997; Corsetti et al., 1999).

A line of research related to the above currency crises strand attempts to produce early warning signals for currency crises. For example, Kaminsky, Lizondo and Reinhart (1998) use an indicator approach whereby crises are signaled when certain macro-financial variables pass specified thresholds. They identify exports, deviations of the real exchange rate from the trend, ratios of broad money to gross international reserves, output growth and equity prices as leading indicators of currency crisis 24 months ahead. In a probabilistic framework, Eichengreen et al. (1996); Frankel and Rose (1996); Berg and Pattillo (1999) employ static probit models and Candelon, Dumitrescu and Hurlin

(2012) estimate dynamic probit models, while Kumar et al. (2003) employ logit models to predict currency crises. All of these studies treat the incident, a crisis, as a dummy variable. The indicator variable is constructed by establishing a threshold that must be crossed by some market pressure index or by the devaluation of the currency. Our study is related to this line of research. However, rather than restricting our analysis to crisis periods, we consider a broader view and examine excessive volatility in the exchange rate, which include crises. Moreover, rather than employing an exogenous threshold for crisis incidence, we opt for a non-parametric pattern recognition algorithm that has widely been used in the business and financial cycles literature.

3.3 Model and relevant variables

3.3.1 Model

We employ an asset price view of the exchange rate; therefore, like any other asset price, e.g., equity prices, exchange rate movements should also be driven by expectations about economic conditions in two countries. Moreover, we argue that, like other asset prices, e.g., equity prices, relative currency prices exhibit cyclical patterns viz., bulls and bears. These cyclical episodes are essentially binary events; therefore, we select a binary choice modeling framework, the probit model. These cycles are revealed only through signals, and the underlying process necessarily remains unobserved. This relationship can be described as follows:

$$S_t^* = \alpha + \sum_{h=0}^{q} \beta_h \widetilde{X}_{t-h} + u_t, \quad \text{where} \quad u_t \sim IID(0, \sigma^2). \tag{3.1}$$

Now, because S_t^* is unobserved, the following censoring is employed to create the binary variable:

$$S_t = \begin{cases} 1, & \text{if} \quad S_t^* > 0, \\ 0, & \text{if} \quad S_t^* \leq 0. \end{cases} \tag{3.2}$$

Letting u_t follow a standard normal distribution results in the probit model, i.e.,

$$Pr(S_t = 1) = \Phi(\alpha + \sum_{h=0}^{q} \beta_h \widetilde{X}_{t-h}). \tag{3.3}$$

In the above specification, S_t represents the exchange rate cycles and $\widetilde{X}_{t-h}$ contains the exogenous macro-financial variables (risk factors) as cross country differences (home *minus* foreign). In addition to this simple model, we note that asset price cycles tend to persist. In the exchange rate market, this persistence reflects the traders' view that "the exchange rates go up by stairs and come down by elevator" (Brunnermeier et al., 2008)! Therefore, we account for this serial dependence by adding the lagged dummy as an exogenous variable. The resulting dynamic model is

$$Pr(S_t = 1) = \Phi(\alpha + \sum_{h=0}^{q} \beta_h \widetilde{X}_{t-h} + \gamma S_{t-1}), \tag{3.4}$$

which can be considered an information augmented model.

We evaluate the in-sample and the out-of-sample (OOS) predictability via the pseudo-R^2, as suggested in Estrella (1998), and via the area under Receiver-Operating-Characteristic curve (AUC), respectively. The AUC as a measure of OOS is widely used in the medical field to verify the accuracy of diagnostic tests. Specifically, the ROC Curve is the scatter plot of 1-*specificity* against the *sensitivity*[2] over a range of cut-off values, $c \in (0, 1)$. It allows discrimination between correct and incorrect classifications. The concavity indicates the extent of correct classifications of bears by the model. Hence, the area under the curve provides an indication of the models ability to discriminate between hits and false alarms. The AUC ranges between 0 and 1, with higher values implying a better model. A value of 0.5, which is just the expectation of the area under the ROC, implies a random model. Hence, an AUC value higher than 0.5 would be interesting.

3.3.2 Identifying Bulls and Bears

Because we employ a binary choice modeling framework, we need to identify the currency bull and bear periods (S_t). Previous applications of the binary choice model focus on forecasting currency crises. Various methods have been employed to locate these crisis episodes. For example, Eichengreen et al. (1996) and Candelon, Dumitrescu and Hurlin (2012) utilize an exchange market pressure index, which is the weighted sum of several relevant macro-financial variables. Eichengreen, Rose, Wyplosz, Dumas and Weber (1995) and Frankel

[2]The so-called *hit rate*, defined as the proportion of bears correctly predicted, is called the *sensitivity*, whereas the proportion of 'no bears' correctly predicted as 'no bears' is called the *specificity* of the model.

and Rose (1996) employ a25% depreciation of currency as the crisis indicator, while Falcetti and Tudela (2006) employ a threshold of 10-20%.Berg and Pattillo (1999) extract the crisis periods from the indicator variables proposed in Kaminsky et al. (1998), while Kumar et al. (2003) conceptualize crashes as large market moves adjusted for interest rate differentials. While these methods identify the crisis episodes, the thresholds are chosen subjectively and they focus on extreme events. However, because we employ the asset view of exchange rates, we will note sustained depreciation lasting for weeks or months and interrupted by intermediate ups and downs. This pattern mirrors equity market bulls and bears, which are characterized by periods of generally increasing or decreasing market prices (see, e.g., Chauvet and Potter, 2000b). To identify similar episodes in the currency market, we opt for Bry and Boschan (1971)'s nonparametric algorithm, which is commonly employed in the business and financial cycle literature. The algorithm recognizes and identifies the patterns of upturns and downturn in a series applying certain censoring criteria. The criteria (rules) correspond to smoothing the data, locating bear (bull) periods within a window surrounding the data point, restricting the minimal phase length and the length of the complete cycle. While such rules for business and equity price cycles are well established (see e.g., Watson, 1994; Pagan and Sossounov, 2003), there are no comparable criteria for exchange rates. Below, we attempt to discern these rules from the existing literature on exchange rate and equity markets as well as from the toolkit of market practitioners.

To identify a currency crisis, Kaminsky and Reinhart (1999) identify a window of 18 months for various macroeconomic variables. Poole (1967) identifies peaks/troughs in the Canadian dollar as movements 0.1-2.0% below or above the previous peak/trough. MacDonald and Young (1986) argue that it is difficult to uniformly apply a threshold for currencies because trending behavior differs from currency to currency. MacDonald and Young, however, classify the local trend if it persists from one month to the next. They classify this trend as either bull or bear. From practitioners' perspective, a 200 day moving average (MA) is widely used to gauge the health of the market, both in currency and equity markets (see e.g., Levich and Thomas III, 1993; Lui and Mole, 1998; Lee et al., 2001; Menkhoff and Taylor, 2007). Various cross-overs (e.g., 5-day MA minus 200 day MA) are used to establish that the market is moving from one phase (say, bull) to the other (bear). Furthermore, Bordo et al. (2001), based on an analysis of 56 countries spanning over 120 years of financial data, report

1.8-2.1 years as the average duration of a currency crisis. Recently, Hutchison and Noy (2005) analyzed 24 emerging markets for banking and currency crises during the period 1975-97 and employ a 24-month window to identify crises. They report an average duration of 1.3 years for a currency crisis.

The foregoing discussion implies a window of 7-18 months, a minimum phase length of one month, and a complete cycle of 15-24 months for the exchange rate market. After comparing timeframes inequity markets, Pagan and Sossounov (2003) suggest a window of 8 months, a phase length of 4 months and a complete cycle of 16 months. Recognizing that exchange rates are also financial assets, we utilize a window length of 7 months. We maintain a smaller window because unlike equity markets, turmoil in the currency market prompts a quicker response (intervention) from the government, which stabilizes the market. Moreover, given the level of trading in this market and global financial integration, deviations of exchange rates induce arbitrage activities, and the resulting misalignment can be corrected by market forces. By the same argument, the span of a complete cycle is set at 15 months, not the 16-month period employed in the equity market. The minimum phase length is however equal to the equity market, i.e., 4 months. Furthermore, following Pagan and Sossounov (2003), we do not smooth the data because this would entail a significant loss of information given the nature of financial time series data.

With the above thresholds, we ensure that peaks and troughs alternate and that the highest peaks and the lowest troughs are selected in cases of multiple peaks or troughs. The location of turning points then amounts to identifying local maxima or minima within a window of $k(=7)$ months. More specifically, a turning point represents a peak at time t if $y_{t-k}, ..., y_{t-1} < y_t > y_{t+1}, ..., y_{t+k}$,while it represents a trough if $y_{t-k}, ..., y_{t-1} > y_t < y_{t+1}, ..., y_{t+k}$. Finally, periods from trough to peak (i.e., *depreciation*[3]) are classified as *bears* $(S_t = 1)$, while periods from peak to trough (i.e., *appreciations*) are classified as *bulls* $(S_t = 0)$.

3.3.3 Explanatory Variables

We now discuss the possible relevant macro-financial variables that might be useful in predicting cycles in the exchange rates. Most previous studies have examined the prediction of currency crises. However, our focus is on the cycles

[3]The exchange rate we examine is the units of domestic currency per unit of foreign currency.

in the exchange rate, which occur during the normal course of business as well as during distress periods. Our identified cycles, therefore, represent the bulls and bears analogous with equity price cycles. While the bear periods represent the turbulent times in the currency market, these periods could include crises episodes as well as periods of high but less severe volatility that may not qualify as crises conditions.

Several studies have noted that the interest rate is considered to be an important determinant of movement of the exchange rates by exchange market practitioners. Frankel and Rose (1996) find that foreign interest rates play an important role in predicting the currency crashes. In a survey of Hong Kong currency dealers, Lui and Mole (1998) report that dealers pay close attention to news about interest rates, especially in the short term. In a similar survey of currency traders in the US, Cheung and Chinn (2001) conclude that even though the importance of particular economic fundamentals varies over time, news about the interest rate remains important. However, we focus on the term spreads and credit spreads in the market rather than the levels of interest rate. The latter have been extensively used in the business and financial cycle literature while the former, which implies liquidity within the market, is considered to be relevant to exchange rate movements by practitioners. Furthermore, exchange rates are theoretically driven by interest rate parity conditions, e.g., uncovered interest rate parity (UIP) across countries. However, empirically, parity conditions do not, at least in the short run (see e.g., Sarno, 2005, for a survey). Therefore, we also consider short-term violations of UIP, which constitute excess returns to currency traders. Additionally, we also consider deviations from another important long run relation, viz., cross-country inflation differentials vis-a-vis exchange rate changes. Moreover, given the globalization and technological advances that have made portfolio investment relatively easy, we also consider how the return differentials on two equity markets affect the exchange rate cycles. Altogether, these variables constitute the risk factors representing a premium due to the term, the credit, the inflation and the interest rate risks. In all cases, we consider the cross-country differentials of the variables, i.e., the *home minus foreign* country variables. Following the exchange rate literature, variables with a star represent foreign variables. The economic intuition and expected behavior of these (differential) variables for exchange rate cycles are discussed below.

Excess Return on Carry Trade (EROCT): Under uncovered interest rate parity condition, the interest rate differential, $(r_t - r_t^*)$, between two countries should be equal to the expected change in the exchange rate between two countries, Δs_{t+1}^e, i.e.,

$$r_t - r_t^* = \Delta s_{t+1}^e, \tag{3.5}$$

wheres_t is the log of nominal exchange rate[4]and $\Delta s_{t+1}^e = Es_{t+1} - s_t$ is the expected depreciation of local currency.

In terms of the UIP relation (3.5), as the interest rate differential increases, the currency with a higher interest rate is expected to depreciate (Krugman et al., 2012). However, empirical evidence suggests that UIP does not hold, especially in the short-run (see e.g., Froot, 1990; Flood and Rose, 2001; Chaboud and Wright, 2005). If UIP does not hold, an investor can earn abnormal returns by borrowing at a lower interest rate and investing at a higher interest rate - the carry, i.e.,

$$E_t z_{t+h} = (r_t^h - r_t^{h*}) - (Es_{t+h} - s_t), \tag{3.6}$$

where $E_t z_{t+h}$ is the expected return at time t on carry trade over horizon h, viz., borrowing in foreign currency (“funding currency”) and investing in local currency ("target currency") for h months. Under UIP $E_t z_{t+h} = 0$, (3.6) is in essence the return in excess of the UIP prediction or the risk premium for exposure to interest rate risk. Cavallo (1984) reports that swings of the dollar against major currencies - JPY and EURO - are due to carry trades, whereby changes in demand and supply conditions, prompted by the opportunity to exploit interest rate differentials, result in sizable and persistent exchange rate movements. The carry trade is a major activity in the foreign exchange market (e.g. Brunnermeier et al., 2008; Christiansen et al., 2011). Consequently, we investigate whether abnormal (excess) returns on carry trades can explain the cyclical behavior of bilateral exchange rate. Specifically, we examine the return on investing in domestic currency by borrowing in foreign currency. It is expected that any increase in excess returns leads to excess demand for the domestic currency, which leads to its appreciation.

Because interest rates exhibit cross-sectional as well as time variation, like(3.6), we also form following switching strategy where the funding currency is always the one with a lower interest rate and the target currency is always the

[4]Throughout, we define exchange rate as the units of domestic currency per unit of foreign currency, the US dollar. Hence, an increase in the rate represents a depreciation of the domestic currency.

one with a higher interest rate (see e.g, Brock and Hommes, 1997, 1998). Therefore, depending on whether the interest rate differential between domestic and foreign country, $dr_t^h = r_t^h - r_t^{h*}$, is positive or negative, the expected excess return, using (3.5), is given by:

$$E_t z_{sw,t+h} = \begin{cases} r_t^h - r_t^{h*} - \Delta s_{t+h}^e & \text{if} \quad dr_t^h > 0, \\ r_t^{h*} - r_t^h + \Delta s_{t+h}^e & \text{if} \quad dr_t t^h < 0, \end{cases} \tag{3.7}$$

where z_{sw} indicates excess return from a switching strategy. The expected movement is appreciation of the high interest rate currency and depreciation of the low interest rate currency (Brunnermeier et al., 2008).

TED Spread: The TED spread is the difference between the interbank rate (IBR), usually the 3-month LIBOR, and the yield on short-term government securities, usually the 3-month maturity T-Bill rate (TB3). The relative TED differential between home and foreign countries is given by,

$$\begin{aligned} dTDS_t &= (TED_t - TED_t^*) \\ &= (IBR_t - TB3_t) - (IBR_t^* - TB3_t^*) \\ &= (IBR_t - IBR_t^*) - (TB3_t - TB3_t^*). \end{aligned} \tag{3.8}$$

As seen from the above relation, a country's yawning TED spread implies credit riskiness vis-a-vis the risk free rate. Because carry trades constitute a significant portion of foreign exchange activities, the TED spread also implies the availability of liquidity to finance such trading via interbank borrowing. This in turn makes the TED spread an indicator of funding liquidity pressures. Because liquidity is a priced factor (Acharya and Pedersen (2005) and Brunnermeier and Pedersen (2009)), its effect via carry trades will be reflected in the exchange rate. When there are funding pressures in the market, positions on carry trade are likely to be unwound (reversed). This distress scenario will result in excess supply of the target currency and an excess demand for the funding currency, leading to the depreciation of the former and a corresponding appreciation of the latter. (see Brunnermeier et al. (2008); Ranaldo and Söderlind (2010); Menkhoff et al. (2011) for a funding liquidity view and Papell and Molodtsova (2012) for a financial distress perspective.)

Relative Purchasing Power Parity (RPPP): The RPPP states that the percent change in the exchange rate of two currencies over any period is equal to the difference between the percent changes in national price levels. The PPP is an important long run relation in foreign exchange. As argued in Krugman et al. (2012), the RPPP is an important relationship because it can be valid even when the absolute PPP is not. Prices are generally sticky, and departures from PPP are greater in the short run than in the long run. Therefore, we explore how these departures affect domestic currency movements. Because deviations originate from the differences in cross-country inflation rates, we expect the differential to be *positively* related to the depreciation cycles.

Relative Stock Market Return (RSMR): There are two approaches to examining the relationship between the stock market and the exchange rate market. According to the traditional approach, changes in exchange rates can change the stock prices of multinational firms directly and those of domestic firms indirectly (Aggarwal, 1981). This view holds that depending on whether a firm is an exporter or a heavy user of imported inputs, the changes in the exchange rates will be reflected in a firm's profits/losses, which will ultimately result in changes in its stock prices. However, the integration of global markets and the ease of capital mobility produce changes in stock prices that could also affect exchange rates via wealth effects or portfolio re-balancing (Bahmani-Oskooee and Sohrabian, 1992). There could, therefore, be a *bi-directional* causality between stock market and exchange market movements (see, Granger et al., 2000). One way of incorporating this insight into our framework is to replace the commodity price index (inflation) with the stock price index (returns) in the PPP (Relative PPP) relation. This view corresponds to Roll (1979)'s efficient market version of the purchasing power parity condition. Incidentally, this relation can also be observed in the UIP context, with equity market returns replacing the bond market returns. Therefore, a positive deviation from this pseudo-parity condition implies the flight of portfolio investment from the home country to the foreign country because of high returns in the foreign equity market and a consequent depreciation of the home currency. A negative relation indicates an influx of portfolio investment in the home country and resulting appreciation of the home currency.

Term Spread : Term spread is the difference between the yields of long term and short-term government securities. The yield curve contains information about expected future economic dynamics and has been found to be a quite robust predictor of business cycles (e.g., Estrella and Hardouvelis, 1991b; Stock and Watson, 1993; Estrella and Mishkin, 1998), stock market cycles (e.g., Chen, 2009; Candelon, Ahmed and Straetmans, 2012) and inflation (e.g., Mishkin, 1990, 1991). The link between the term spread and exchange rate has, however, hardly been explored empirically. Recently, Chen and Tsang (2013) have explored how three Nelson-Seigel factors[5] of term structure relate to the exchange rate changes. We contribute to this burgeoning research area and employ the term spread to verify its usefulness for predicting the cycles in the exchange rate. Let R_t and R_t^* be the yields on home and foreign government long-term bonds and r_t and r_t^* be the corresponding yields on short-term government paper. The term spread differential is:

$$\begin{aligned} dSP_t &= (R_t - r_t) - (R_t^* - r_t^*) \\ &= (R_t - R_t^*) - (r_t - r_t^*) \\ &= \underbrace{(R_t - R_t^*)}_{A} - \underbrace{\Delta s_{t+1}^e}_{B}, \end{aligned} \tag{3.9}$$

where the third equality above employs the UIP relation (3.5). The link can be viewed from two perspectives. First, movements at the longer end of the yield curve are mainly explained by changes in expected inflation (Mishkin, 1991). Therefore, a widening relative gap, according to the expectations hypothesis, implies higher expected domestic inflation (part A in 3.9 above)and leads to expected depreciation of the domestic currency in the long run. In the short-run, however, the domestic currency may either appreciate ('overshooting'), as implied by third equality (part B in 3.9), or depreciate if the increase in interest rates is considered by the market to be due to a higher expected inflation premium (Mishkin and Eakins (2012); Krugman et al. (2012)). Second, a relative rise in the term spread indicates an improving domestic economic outlook. This improvement may decrease riskiness and produce a corresponding fall in the risk premium and an appreciation of domestic currency. The domestic currency may, however, later depreciate as strong exchange rate signals lower exports and output. Depending on which effect (inflation or risk premium) is dominant, an increase in the relative term spread may have either an appreciating or a depreciating impact on the domestic currency. The behavior of term spread is therefore ambiguous.

[5]The three term structure factors are the level, the slope and the curvature (Nelson and Siegel (1987)); they are conjectured to reflect the long term inflation expectations, business cycle and the central bank's monetary stance, respectively Dewachter and Lyrio (2006); Rudebusch and Wu (2008).

3.4 Empirical Results

In this study, we examine six exchange rates viz., Australian dollar (AUD), Euro (EUR), Japanese yen (JPY), New Zealand dollar (NZD), Swiss franc (CHF), and the British pound (GBP) against the United States dollar (USD). The exchange rate is the units of home currency per unit of foreign currency, which is the US dollar. These exchange rates represent a mix of reserve, funding and investment currencies and are the most traded in terms of daily market turnover representing approximately 82% of average daily turnover (see Table B.4 on p-12 in BIS, 2010). The monthly data spans from January 1980 to December 2012, except for EUR, which starts from January 1999. The explanatory risk factors, viz., the TED spread, the relative PPP and the term spread are available for the full sample period; however, due the availability of LIBOR rates from the mid-80s, the excess returns on carry trade (EROCT) are from February 1986 to December 2012. Moreover, because LIBOR rates are unavailable for longer periods, we have employed 3-month money market rates for the calculation of TED spread. The data have been obtained from various databases, including the IMF's International Finance Statistics (IFS), and national sources via Datastream, the FRED (FRB St. Louis) and the OECD (see Table 3.10 in appendix for details). Because we study the behavior of bilateral exchange rates, the US dollar is treated as foreign currency and we take the cross country differential of the relevant exogenous variables. For example, the term spread utilized in the model is actually the difference between the domestic term spread and the corresponding foreign term spread. The cycles, i.e., the bull and the bear episodes, have been determined via the Bry and Boschan (1971) algorithm (BBA) using the censoring criteria elaborated in section (3.3.2). Figure 3.1 displays the historical behavior of exchange rates along with the cycles extracted via BBA. The shaded areas represent the periods of home currency depreciation (bear periods). As can be observed, the algorithm adequately captures these cyclical episodes. For example, the prolonged depreciation of the Swiss franc from the early 1980s to the mid-1980s is quite remarkable. This depreciation reflects the consequences of the efforts of the Swiss National Bank to reverse the course of sustained currency appreciation after the franc was floated in 1973 (Rich, 1990) as well as the corresponding appreciation of the US dollar during Paul Volcker's inflation control via tight monetary policy (see e.g., Frankel et al., 1994). The consequent depreciation of almost all currencies against the dollar due to US monetary policy is quite visible. Looking at the comparative movement of exchange rates, it is obvious that two of the major funding currencies, i.e., the Japanese yen and the Swiss franc, as well as the euro have shown marked appreciation against the Greenback, but the investment currencies, such as the Australian and New Zealand dollars, have behaved in the opposite manner, especially during the early part of the sample period. After the sub-prime mortgage crisis in the US, however, the US dollar has largely depreciated against these currencies, except the British Pound and the euro. Post crisis, there are on average only 2-3 episodes of depreciation the currencies against the US dollar. As we shall discuss below, this has implications for the currency carry trade.

Table 3.1 provides the summary statistics of changes in bilateral exchange rates (panel A) and the exchange rate cycles (panel B). In panel A of the table, we report the mean (μ), the volatility (σ), the skewness and the kurtosis of exchange rate returns alongwith their autocorrelations (ρ) over one month, one quarter and two quarters. All the statistics are based on annualized returns. Over the sample period, funding currencies, on average, have appreciated while the investment currencies have depreciated, as previously noted. Moreover, currencies with negative average returns tend to be negatively skewed, implying that though these currencies could yield frequent positive gains, these are also exposed to the downside risk. In concrete terms, this means that by investing in the US dollar by borrowing in the Japanese yen (JPY), for example, one can earn the cross-country interest rate differential plus the 3.17% depreciation of the JPY but would simultaneously be exposed to negative skewness (currency crash risk) of 0.57 (Brunnermeier et al., 2008). Interestingly, the annualized volatility of all the currencies is similar (approximately 30-35%) and is comparable to that of equity markets. The currencies subject to carry trade are on average more volatile, however. Except for the euro, other exchange rates exhibit a kurtosis higher than the normal distribution, implying fat tails. This leptokurtic distribution of currency returns points to the fact that although there is a high likelihood that the returns cluster around the center of distribution, the chances of earning abnormal returns and large losses remain equally probable. This is more characteristic of the investment currencies, i.e., those offering high interest rates (e.g., AUD, NZD), which tend to have a more peaked return distribution and are positively skewed[6]. Furthermore, currency returns

[6]Between August 2008 and October 2008, the VIX index, which gauges the risk sentiment of market, rose from a trough of 19 to a peak of 80, leading to extreme volatility in the market. A typical investment currency lost 2.6 years of yield advantage during that brief period of tumult (McCauley and McGuire, 2009)

exhibit momentum for at least a quarter, as implied by positive autocorrelation over one and three months (see ρ_1 and ρ_3). There is, however, a mean reversion after at least two quarters as the ρ_6 becomes negative.

Table 3.1: Summary Statistics for Changes in FX Rate (Δs_t) and the FX Cycles (S_t)

	Panel A: Δs_t							Panel B: FX Cycles			
	μ	σ	Skew	Kurt	ρ_1	ρ_3	ρ_6	D_{TP}	A_{TP}	D_{PT}	A_{PT}
AUD	0.18	32.2	1.08	7.7	0.35	0.02	-0.01	19	0.21	19	-0.23
EUR	-0.89	30.5	-0.01	3.0	0.30	0.01	0.02	11	0.27	26	-0.21
JPY	-3.17	32.9	-0.57	3.9	0.30	0.02	-0.13	15	0.27	22	-0.19
NZD	0.52	33.8	0.53	5.5	0.34	0.13	0.07	20	0.26	19	-0.28
CHF	-1.67	35.3	-0.07	3.6	0.27	0.04	-0.01	21	0.32	21	-0.24
GBP	1.03	30.0	0.27	4.7	0.33	0.09	-0.08	18	0.18	19	-0.23

Note: i. Monthly data from 1980M1 to 2012M12. ii. FX rate is the exchange rate in units of domestic currency per unit of foreign currency (the US Dollar). iii. $\Delta s_t [= 1200 \times (s_t - s_{t-1})]$ is the annualized percentage changes in the log exchange rates (s_t). iv. D is average phase duration of THE FX cycle and A is its corresponding average amplitude. v. TP is trough-to-peak or depreciation phase (bear) and PT is peak-to-trough or appreciation phase (bull).

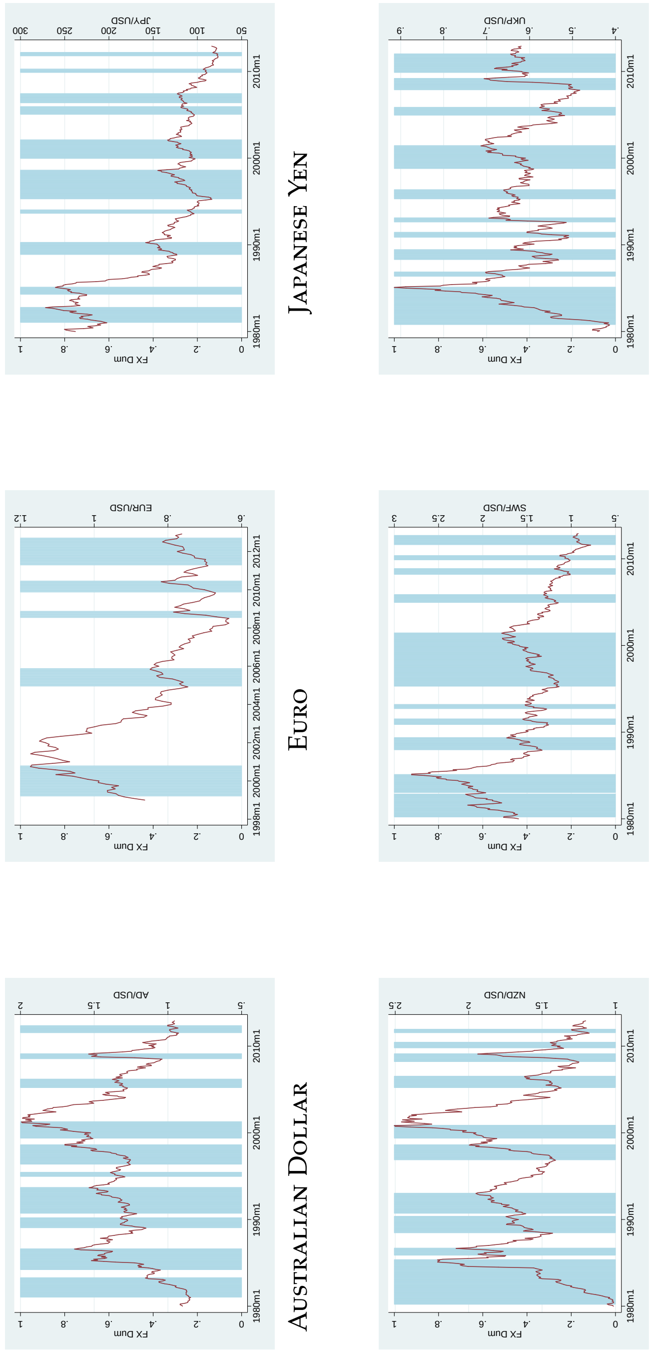

Figure 3.1: Exchange Rate Cycles v/s Exchange Rates - FX Rate as units of domestic currency per US Dollar

Panel B of Table 3.1 lists the major characteristics of the exchange rate cycles. We report the average phase duration (D) and the amplitude (A). 'D' represents the number of months spent in a particular state (bull or bear), while 'A' shows the average price change from one turning point to the next, i.e., from peak to trough or vice-versa. The average durations of bear and bull states are similar for four of the six currencies, ranging between 18-19 months. For the euro and the JPY, however, periods of appreciation last longer than the corresponding periods of depreciation over the sample period. This pattern is consistent with the general behavior of these currencies against the Greenback, as is also evident from Figure 3.1. Incidentally, as implied by the amplitude (A), the average gains and losses to a typical currency trader during the weakening or strengthening of currencies are remarkably similar.

Having examined the summary statistics, we now discuss the estimation results from the simple and the dynamic specifications of the probit models. We estimate various models each including lagged explanatory variables with lags at, $h = 1, 3, 6, 9, 12, 15, 18, 21,$ and 24. We report only the parameter estimates on the explanatory variable, $\widehat{\beta}$, alongwith $p-$values based on White's robust sandwich estimator for the asymptotic covariance matrix. We also compare both in-sample and the out-of-sample performance of all the models via Estrella (1998)'s goodness of fit measure, pseudo$-R^2$, and the Area under ROC curve.

The first variable for explaining the currency depreciation cycles is the excess return on carry trade (EROCT). As argued previously, the carry trade has been found to be a significant activity in the currency exchange market and has recently attracted considerable interest from researchers (see e.g., Brunnermeier et al., 2008; Jorda and Taylor, 2012; Menkhoff et al., 2011; Spronk et al., 2013). Tables 3.2 and 3.3 contain the estimation results. We report results for excess returns *without* and *with* the switching strategy in two tables. While the excess return represents a violation of interest rate parity conditions, these can also be conceptualized as the yield on an exposure to a portfolio containing currencies. A portfolio *without* switching is analogous to a long position on the domestic interest rate and a short position on the foreign interest rate. A portfolio *with* a switch exploits cross-country interest rate differentials and takes a long position in currencies offering high interest rates and takes a short position on a currency with a low interest rate. The estimation results from the simple probit model including EROCT without switching, z_{t+m}, in Table 3.2

Table 3.2: PROBIT MODEL ESTIMATION RESULTS WITH EXCESS RETURN ON CARRY TRADE (EROCT) - z_{t+m} - NO SWITCHING

h	**1**	**3**	**6**	**9**	**12**	**1**	**3**	**6**	**9**	**12**
	SIMPLE PROBIT					DYNAMIC PROBIT				
	IN-SAMPLE RESULTS									
	$\widehat{\beta}$					$\widehat{\beta}$				
AUS	-0.023	-0.051	-0.065	-0.060	-0.051	-0.027	-0.012	0.000	0.005	0.018
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.027***	*0.966*	*0.629*	*0.039*
EU	-0.027	-0.053	-0.056	-0.048	-0.038	-0.034	-0.010	0.007	0.033	0.034
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.316*	*0.493*	*0.054*	***0.003***
JP	-0.024	-0.041	-0.058	-0.077	-0.089	-0.031	-0.005	0.006	-0.003	-0.016
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.480*	*0.358*	*0.794*	*0.256*
NZ	-0.023	-0.041	-0.037	-0.019	-0.010	-0.025	-0.010	0.010	0.020	0.025
	0.000	***0.000***	***0.000***	***0.025***	*0.274*	***0.000***	*0.103*	*0.236*	*0.075*	*0.073*
SWT	-0.022	-0.040	-0.047	-0.048	-0.048	-0.026	-0.003	0.031	0.032	0.027
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.672*	***0.001***	***0.001***	***0.011***
UK	-0.024	-0.046	-0.054	-0.041	-0.039	-0.042	-0.004	0.011	0.029	0.030
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.388*	*0.129*	***0.005***	***0.006***
	R-SQUARED					R-SQUARED				
AUS	0.225	0.359	0.367	0.288	0.196	0.866	0.817	0.810	0.810	0.813
EU	0.284	0.403	0.313	0.179	0.088	0.864	0.773	0.759	0.761	0.748
JP	0.238	0.317	0.338	0.377	0.403	0.869	0.802	0.801	0.799	0.802
NZ	0.300	0.382	0.246	0.062	0.013	0.805	0.714	0.710	0.722	0.722
SWT	0.247	0.342	0.276	0.200	0.160	0.879	0.819	0.835	0.830	0.824
UK	0.202	0.272	0.241	0.122	0.081	0.866	0.788	0.799	0.807	0.813
	AREA UNDER ROC					AREA UNDER ROC				
AUS	0.773	0.830	0.843	0.814	0.760	0.983	0.957	0.947	0.939	0.949
EU	0.807	0.858	0.823	0.756	0.673	0.980	0.945	0.944	0.952	0.941
JP	0.786	0.828	0.828	0.837	0.848	0.980	0.942	0.947	0.946	0.961
NZ	0.843	0.858	0.786	0.665	0.570	0.979	0.931	0.932	0.939	0.949
SWT	0.778	0.829	0.800	0.752	0.720	0.982	0.950	0.964	0.964	0.958
UK	0.742	0.788	0.789	0.735	0.673	0.979	0.944	0.950	0.954	0.961

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$ is the EROCT, the explanatory variable. $\widetilde{X}_{t-k}$ is the relative difference between the domestic and the foreign EROCT (z_{t+m}). EROCT without and with switching are given by eqn. (3.6) and (3.6), respectively. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1986M2-2012M12.

demonstrate that UIP violations are highly statistically significant predictors of the depreciation cycles for a period of one year[7]. As expected, increasing excess returns increase the probability of appreciation. The relationship is stable even when the persistence of cycles is taken into consideration via a dynamic probit model. Nonetheless, once the information set contains the lagged cyclical phase, the excess return becomes a short-term phenomenon. This change reflects the fact that any opportunities to exploit violations of UIP in the market

[7]Due to the unavailability of LIBOR rates for maturities corresponding to our preferred forecast horizons (lag lengths), i.e., $h = 1, 3, 6, 9, 12, 15, 18, 21, and$ 24, we only estimate the models with $h = 1, 3, 6, 9, and$ 12.

Table 3.3: PROBIT MODEL ESTIMATION RESULTS WITH EXCESS RETURN ON CARRY TRADE (EROCT) - $z_{sw,t+m}$ - WITH SWITCHING

h	**1**	**3**	**6**	**9**	**12**	**1**	**3**	**6**	**9**	**12**
	SIMPLE PROBIT					DYNAMIC PROBIT				
	IN-SAMPLE RESULTS									
	$\widehat{\beta}$					$\widehat{\beta}$				
AUS	-0.014	-0.027	-0.042	-0.047	-0.054	-0.019	-0.008	-0.002	-0.003	0.007
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.135*	*0.763*	*0.766*	*0.577*
EU	-0.006	-0.008	-0.004	0.000	0.002	-0.013	-0.003	0.014	0.014	0.008
	0.061	*0.101*	*0.594*	*0.999*	*0.826*	***0.040***	*0.712*	*0.123*	*0.330*	*0.650*
JP	0.020	0.034	0.047	0.062	0.079	0.024	0.004	-0.012	-0.003	0.020
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.574*	*0.098*	*0.757*	*0.120*
NZ	-0.023	-0.041	-0.037	-0.019	-0.010	-0.025	-0.010	0.010	0.020	0.025
	0.000	***0.000***	***0.000***	***0.025***	*0.274*	***0.000***	*0.103*	*0.236*	*0.075*	*0.073*
SWT	0.011	0.016	0.027	0.046	0.053	0.008	0.000	0.001	0.010	-0.001
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.030***	*0.917*	*0.875*	*0.386*	*0.925*
UK	-0.021	-0.036	-0.043	-0.034	-0.030	-0.034	-0.002	0.011	0.028	0.028
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.609*	*0.128*	***0.004***	***0.006***
	R-SQUARED					R-SQUARED				
AUS	0.095	0.143	0.184	0.181	0.177	0.846	0.811	0.807	0.804	0.802
EU	0.021	0.017	0.002	0.000	0.000	0.798	0.768	0.762	0.734	0.755
JP	0.177	0.229	0.242	0.274	0.338	0.851	0.800	0.820	0.797	0.801
NZ	0.300	0.382	0.246	0.062	0.013	0.805	0.714	0.710	0.722	0.722
SWT	0.076	0.075	0.111	0.178	0.181	0.828	0.813	0.814	0.812	0.807
UK	0.165	0.208	0.183	0.090	0.052	0.850	0.778	0.784	0.792	0.795
	AREA UNDER ROC					AREA UNDER ROC				
AUS	0.687	0.716	0.738	0.756	0.748	0.973	0.951	0.950	0.942	0.946
EU	0.575	0.563	0.514	0.500	0.488	0.961	0.938	0.936	0.953	0.942
JP	0.744	0.779	0.786	0.795	0.824	0.973	0.940	0.961	0.948	0.957
NZ	0.843	0.858	0.786	0.665	0.570	0.979	0.931	0.932	0.939	0.949
SWT	0.672	0.668	0.701	0.737	0.735	0.964	0.948	0.951	0.943	0.943
UK	0.721	0.747	0.743	0.704	0.625	0.978	0.937	0.947	0.950	0.955

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$ is the EROCT, the explanatory variable. $\widetilde{X}_{t-k}$ is the relative difference between the domestic and the foreign EROCT (z_{t+m}). EROCT without and with switching are given by eqn. (3.6) and (3.6), respectively. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1986M2-2012M12.

quickly evaporate. As a first indication of cyclical behavior of exchange rates, the model predicts a reversal (depreciation) of Swiss franc and British pound over three to four quarters. The in-sample fit (R^2) of the simple probit model is reasonably high and increases 4-7 times under the dynamic specification. The AUC measure of out-of-sample performance corroborates the in-sample fit.

Table 3.4: Probit Model Estimation Results with Relative TED Spread

h	1	3	6	9	12	15	18	21	24	1	3	6	9	12	15	18	21	24
	Panel A: Simple Probit									Panel B: Dynamic Probit								
	In-sample Results																	
	$\widehat{\beta}$									$\widehat{\beta}$								
AUD	-0.051	-0.129	-0.151	-0.017	0.065	0.093	0.134	0.230	0.215	-0.241	-0.227	0.000	0.167	0.011	0.088	0.015	0.225	-0.089
	0.600	*0.183*	*0.131*	*0.858*	*0.502*	*0.337*	*0.170*	***0.023***	***0.030***	*0.134*	*0.254*	*1.000*	*0.348*	*0.948*	*0.569*	*0.908*	*0.061*	*0.411*
EUR	-0.224	0.133	0.202	0.085	-0.039	-0.217	-0.091	0.363	0.377	0.225	0.427	0.180	-0.110	-0.106	-0.110	0.102	0.383	-0.114
	0.250	*0.456*	*0.284*	*0.669*	*0.847*	*0.290*	*0.689*	*0.203*	*0.186*	*0.480*	*0.413*	*0.568*	*0.729*	*0.786*	*0.767*	*0.632*	*0.201*	*0.747*
JPY	-0.209	-0.237	-0.292	-0.344	-0.384	-0.456	-0.409	-0.312	-0.230	-0.107	-0.109	-0.180	-0.200	-0.247	-0.203	-0.127	-0.054	-0.034
	0.035	***0.015***	***0.002***	***0.000***	***0.000***	***0.000***	***0.000***	***0.001***	***0.008***	*0.368*	*0.466*	*0.171*	*0.109*	*0.057*	*0.218*	*0.364*	*0.711*	*0.807*
NZD	0.303	0.364	0.379	0.245	0.157	0.147	0.181	0.239	0.254	0.193	0.305	0.153	0.003	0.000	0.092	0.112	0.171	0.060
	0.000	***0.000***	***0.000***	***0.000***	***0.002***	***0.003***	***0.000***	***0.000***	***0.000***	***0.019***	***0.020***	***0.050***	*0.967*	*0.999*	*0.252*	*0.147*	***0.044***	*0.461*
CHF	-0.607	-0.688	-0.838	-0.643	-0.513	-0.416	-0.140	-0.167	-0.229	-0.521	-0.471	-0.154	-0.164	0.110	0.295	0.164	-0.405	-0.147
	0.000	***0.000***	***0.000***	***0.000***	***0.001***	***0.006***	*0.337*	*0.245*	*0.114*	*0.062*	*0.134*	*0.537*	*0.394*	*0.681*	*0.179*	*0.544*	*0.164*	*0.629*
GBP	-0.816	-0.897	-0.966	-0.778	-0.652	-0.620	-0.542	-0.343	-0.403	-0.699	-0.334	-0.335	-0.024	-0.145	-0.224	-0.180	0.046	-0.412
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.001***	***0.031***	***0.012***	***0.021***	*0.294*	*0.268*	*0.917*	*0.550*	*0.388*	*0.444*	*0.852*	*0.145*
	R-squared									R-squared								
AUD	0.001	0.004	0.006	0.000	0.001	0.002	0.005	0.014	0.013	0.824	0.823	0.819	0.819	0.816	0.824	0.822	0.823	0.820
EUR	0.007	0.002	0.005	0.001	0.000	0.007	0.001	0.015	0.016	0.781	0.777	0.763	0.750	0.737	0.721	0.704	0.694	0.702
JPY	0.017	0.021	0.031	0.043	0.053	0.070	0.058	0.036	0.020	0.803	0.803	0.804	0.804	0.805	0.803	0.800	0.798	0.798
NZD	0.086	0.109	0.114	0.068	0.034	0.030	0.043	0.068	0.073	0.826	0.830	0.832	0.827	0.825	0.827	0.826	0.828	0.820
CHF	0.043	0.054	0.077	0.048	0.032	0.021	0.002	0.003	0.007	0.829	0.826	0.829	0.827	0.826	0.826	0.823	0.825	0.820
GBP	0.060	0.071	0.081	0.055	0.039	0.036	0.028	0.012	0.016	0.828	0.821	0.819	0.816	0.825	0.824	0.821	0.819	0.820
	Out-of-sample Results																	
	Area Under ROC Curve									Area Under ROC Curve								
AUD	0.499	0.546	0.559	0.500	0.500	0.511	0.530	0.560	0.553	0.955	0.958	0.949	0.951	0.946	0.953	0.950	0.956	0.957
EUR	0.578	0.445	0.531	0.509	0.495	0.517	0.507	0.650	0.613	0.947	0.942	0.942	0.933	0.942	0.942	0.928	0.941	0.939
JPY	0.659	0.645	0.618	0.625	0.627	0.635	0.606	0.571	0.542	0.948	0.951	0.951	0.954	0.952	0.946	0.943	0.947	0.948
NZD	0.522	0.526	0.522	0.524	0.541	0.549	0.561	0.533	0.519	0.956	0.955	0.950	0.953	0.951	0.957	0.956	0.958	0.952
CHF	0.627	0.645	0.676	0.635	0.630	0.588	0.508	0.520	0.528	0.963	0.958	0.956	0.951	0.953	0.952	0.952	0.959	0.950
GBP	0.623	0.634	0.641	0.610	0.586	0.578	0.567	0.544	0.548	0.962	0.952	0.956	0.949	0.954	0.956	0.954	0.949	0.955

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta\widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$ is relative TED spread, the explanatory variable. $\widetilde{X}_{t-k}$ is the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

The results including EROCT *with* a switching strategy, $z_{sw,t+m}$, presented in Table 3.3 are consistent with expectation, i.e., the investment currencies are predicted to appreciate while the funding currencies depreciate over the periods considered. As with previous results, EROCT is a highly significant predictor of depreciation cycles in the simple probit specification; however, it can only predict over the short term in the information augmented dynamic probit model. For the GBP, however, an increasing EROCT predicts an appreciation in the short run but a subsequent depreciation over 3-4 quarters. The in- and out-of-sample fits of the models containing EROCT without switching (panel A) are similar to the models with switching (panel B). The fact that the information augmented dynamic probit model predicts depreciation of a funding currency, e.g., CHF, when considered with the negative skewness of the funding currencies, might have important implications for currency traders. While favorable interest rate differentials might be a boon to investors, any adverse movements in the differentials will negate their profits (McCauley and McGuire, 2009; Jorda and Taylor, 2012). These adverse swings may force investors to unwind their positions, putting additional pressure on the investment currencies and amplifying their losses. The losses may invoke margin calls on the short positions of investors and create liquidity pressures in the market, which can ultimately lead to liquidity spirals (Brunnermeier and Pedersen, 2009; Menkhoff et al., 2011). Therefore, to observe how liquidity constraints affect exchange rate movements, we follow Menkhoff et al. (2011) and Papell and Molodtsova (2012) and employ the TED spread, which proxies the credit risk or funding pressure in the market. Table 3.4 reports the results. It is clear from panel A of the table that an increase in the relative TED spread leads to appreciation of the two major funding currencies (JPY and CHF) as well as the British pound. Of the investment currencies, only the NZD responds with depreciation. This pattern clearly suggests that during times of funding pressure in the market, investors are forced to unwind their carry trades whereby they unload their investments in high interest currencies. This creates an excess supply of investment currencies and an excess demand for funding currencies. Consequently, the investment currencies depreciate and funding currencies appreciate, leading to losses on carry trades. Brunnermeier et al. (2008) and Menkhoff et al. (2011) have conjectured that the unwinding of carry trade positions could be driving exceptional movements in the exchange rate, especially in absence of macroeconomic news. Melvin and Taylor

(2009), who describe the TED spread as the market price of credit risk, have ascribed unusually high volatility of the exchange rates to the liquidity constraints faced by currency market investors, especially after the Lehman crisis in late 2008. The estimates based on the dynamic probit model presented in panel B of the table demonstrate that the TED spread exhibits predictability for the NZD for upto two quarters and for GBP for upto one month. For the AUD, the JPY and the CHF, however, it is marginally significant. Persistent depreciation of the NZD, even in the wake of increasing liquidity constraints, implies anchored beliefs of investors due to the prolonged period of high interest rates in New Zealand. However, the predicted behavior in all cases is as expected. Furthermore, although the in- and out-of-sample fits of the simple probit model employing the TED spread are not as good as that of EROCT; the model outperforms in the out-of-sample analysis.

Table 3.5: PROBIT MODEL ESTIMATION RESULTS WITH RELATIVE RPPP DEVIATIONS

h	1	3	6	9	12	15	18	21	24	1	3	6	9	12	15	18	21	24
	PANEL A: SIMPLE PROBIT									PANEL B: DYNAMIC PROBIT								
	IN-SAMPLE RESULTS																	
	$\widehat{\beta}$									$\widehat{\beta}$								
AUD	0.022	0.044	0.067	0.072	0.068	0.066	0.063	0.062	0.061	0.028	0.008	0.004	0.001	-0.012	-0.013	-0.014	-0.008	-0.012
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.149*	*0.611*	*0.876*	*0.156*	*0.264*	*0.224*	*0.535*	*0.396*
EUR	0.028	0.056	0.063	0.055	0.047	0.043	0.049	0.052	0.041	0.034	0.014	-0.001	-0.030	-0.030	-0.023	-0.012	-0.016	-0.027
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.006***	***0.000***	*0.180*	*0.925*	*0.083*	***0.018***	*0.083*	*0.544*	*0.511*	***0.205***
JPY	0.022	0.040	0.059	0.081	0.090	0.092	0.077	0.075	0.069	0.032	0.006	-0.004	0.001	0.011	0.002	-0.010	-0.003	-0.011
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.256*	*0.449*	*0.859*	*0.346*	*0.834*	*0.392*	*0.783*	*0.364*
NZD	0.023	0.050	0.063	0.050	0.044	0.045	0.051	0.056	0.056	0.030	0.019	-0.004	-0.005	-0.012	-0.006	0.002	0.003	-0.002
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.586*	*0.585*	*0.198*	*0.561*	*0.827*	*0.809*	*0.888*
CHF	0.018	0.036	0.047	0.052	0.055	0.051	0.053	0.059	0.062	0.021	0.003	-0.017	-0.017	-0.016	-0.018	-0.018	-0.009	-0.013
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.603*	***0.014***	***0.019***	*0.065*	*0.075*	*0.063*	*0.384*	*0.243*
GBP	0.024	0.050	0.061	0.049	0.050	0.044	0.039	0.042	0.051	0.039	0.008	-0.008	-0.023	-0.021	-0.027	-0.026	-0.012	-0.003
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.101*	*0.235*	***0.012***	*0.036*	*0.039*	*0.101*	*0.454*	*0.838*
	R-SQUARED									R-SQUARED								
AUD	0.202	0.311	0.370	0.341	0.269	0.221	0.177	0.158	0.139	0.874	0.822	0.818	0.817	0.817	0.825	0.824	0.821	0.820
EUR	0.270	0.413	0.350	0.213	0.119	0.073	0.075	0.074	0.041	0.864	0.776	0.758	0.757	0.743	0.721	0.698	0.678	0.707
JPY	0.222	0.307	0.341	0.381	0.382	0.353	0.255	0.219	0.174	0.870	0.803	0.801	0.800	0.801	0.807	0.805	0.801	0.798
NZD	0.236	0.401	0.392	0.276	0.200	0.171	0.182	0.189	0.171	0.896	0.841	0.828	0.827	0.827	0.824	0.822	0.820	0.819
CHF	0.202	0.317	0.295	0.247	0.219	0.170	0.149	0.152	0.148	0.872	0.829	0.834	0.831	0.828	0.827	0.825	0.821	0.820
GBP	0.217	0.328	0.313	0.193	0.151	0.103	0.069	0.068	0.085	0.886	0.820	0.818	0.825	0.829	0.830	0.826	0.819	0.816
	OUT-OF-SAMPLE RESULTS																	
	AREA UNDER ROC CURVE									AREA UNDER ROC CURVE								
AUD	0.757	0.811	0.844	0.838	0.795	0.756	0.731	0.710	0.694	0.985	0.957	0.950	0.953	0.950	0.953	0.944	0.946	0.948
EUR	0.806	0.863	0.839	0.781	0.712	0.642	0.653	0.665	0.628	0.980	0.953	0.940	0.943	0.939	0.929	0.930	0.924	0.932
JPY	0.774	0.819	0.827	0.844	0.845	0.833	0.786	0.759	0.727	0.981	0.947	0.944	0.946	0.951	0.948	0.944	0.947	0.943
NZD	0.786	0.862	0.859	0.819	0.776	0.743	0.738	0.745	0.728	0.986	0.966	0.955	0.948	0.955	0.950	0.945	0.948	0.947
CHF	0.766	0.827	0.807	0.774	0.758	0.731	0.713	0.712	0.706	0.979	0.953	0.961	0.957	0.962	0.957	0.953	0.954	0.953
GBP	0.754	0.816	0.820	0.773	0.724	0.689	0.659	0.646	0.647	0.984	0.950	0.950	0.958	0.956	0.959	0.956	0.957	0.949

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta \breve{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta \breve{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\breve{X}_{t-k}$ is the RPPP, the explanatory variable. $\breve{X}_{t-k}$ is the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

The countries included in this study possess central banks that have been mandated, explicitly or implicitly, to maintain inflation below a target rate. Currencies have generally been found to respond to news about inflation because the loss or gain of purchasing power is reflected in the weakness or strength a currency (see e.g., Engel and West, 2006; Clarida and Waldman, 2008). In addition to interest rate parity, relative purchasing power parity (RPPP) is a theoretically and empirically important relation in international economics. Due to the stickiness of prices, short run deviations of RPPP have important implications for exchange rate movements. Therefore, we also investigate whether these short run deviations are useful predictors of the direction of exchange rate change. Table 3.5 displays the estimation results from the two probit models employing RPPP as an explanatory variable. The $\widehat{\beta}$ estimates from the simple probit in panel A of the table demonstrate that an increase in RPPP deviation, which signifies increasing domestic inflation, can significantly forecast depreciation of the domestic currency. The explanatory power, R^2, is as high as that of EROCT, demonstrating that deviations from the two parity conditions are useful predictors of exchange rate movements. Similarly to the estimates based on dynamic probit with EROCT, once the persistence of cycles is accounted for, the departures from this long run relation are of a short run nature (see panel B of the table under discussion). This is consistent with findings about the UIP and RPPP in the exchange literature (see e.g., Molodtsova and Papell, 2009). Interestingly, a reversal (appreciation) begins after one quarter, which for some currencies, e.g., euro and CHF, is quite significant. Specifically, after depreciating for one quarter (though the depreciation is significant for one month only) , the euro appreciates at one and two year time periods. For the CHF, however, the reversal occurs after one quarter and continues for two quarters, while the reversal occurs at the third quarter for the UK. Comparing the results from two probit models, simple vs. dynamic, the sustained significance of RPPP deviations over all time periods imply the systematic under/overestimation of price shocks by investors. In dynamic models, however, the persistence of cycles is taken into account and a revision of beliefs occurs, which leads to reversals. This pattern is similar to the updating effect of interest rate shocks observed in Gourinchas and Tornell (2004). As a result of agents updating their beliefs, the UIP and RPPP violations become short run phenomena. Out-of-sample, the dynamic probit is outperforms the simple probit, exhibiting a higher value of the AUC.

The purchasing power parity condition can be restated with commodity price index replacing the stock price index as suggested in Roll (1979) under efficient market conditions (see also, Morley, 2002). The relative version of this pseudo-parity condition (where inflation is replaced with equity return) implies both the RPPP as well as UIP conditions. We, therefore, also employ deviations from this pseudo-parity condition and investigate whether these can predict depreciation cycles in the exchange rate. Table 3.6 shows the results. Under the simple probit, significant and positive coefficients point to the dominance of the portfolio view or switching equity investment from domestic to foreign stock market and the consequent depreciation of home currency. Again, the deviations can only predict cycles in the short run once the information augmented dynamic model is employed; reversals are predicted over three to six quarters for GBP.

Table 3.6: Probit Model Estimation Results with Relative Stock Market Return deviations

h	1	3	6	9	12	15	18	21	24	1	3	6	9	12	15	18	21	24
	Panel A: Simple Probit									Panel B: Dynamic Probit								
	In-sample Results																	
	$\widehat{\beta}$									$\widehat{\beta}$								
AUD	0.006	0.019	0.036	0.051	0.051	0.050	0.048	0.052	0.055	0.006	0.010	0.011	0.015	0.001	-0.001	0.002	0.007	0.007
	0.007	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.023***	***0.012***	***0.026***	*0.063*	*0.916*	*0.894*	*0.864*	*0.484*	*0.571*
EUR	0.008	0.015	0.016	0.007	0.003	0.011	0.027	0.040	0.037	0.011	0.006	-0.004	-0.019	-0.017	-0.006	0.006	0.009	-0.003
	0.043	*0.064*	*0.194*	*0.597*	*0.837*	*0.544*	*0.200*	*0.097*	*0.158*	***0.003***	*0.282*	*0.574*	*0.136*	*0.118*	*0.671*	*0.743*	*0.628*	*0.869*
JPY	0.007	0.012	0.017	0.018	0.017	0.014	0.011	0.010	0.010	0.010	0.002	0.000	-0.001	-0.002	-0.005	-0.005	-0.005	-0.004
	0.000	***0.000***	***0.000***	***0.000***	***0.001***	***0.010***	*0.055*	*0.093*	*0.129*	***0.000***	*0.401*	*0.950*	*0.884*	*0.656*	*0.298*	*0.277*	*0.362*	*0.381*
NZD	0.005	0.010	0.014	0.016	0.017	0.018	0.022	0.030	0.037	0.005	0.006	0.000	0.003	0.001	0.002	0.007	0.012	0.014
	0.001	***0.001***	***0.001***	***0.001***	***0.002***	***0.003***	***0.001***	***0.000***	***0.000***	***0.003***	***0.007***	*0.920*	*0.569*	*0.839*	*0.757*	*0.224*	*0.123*	*0.074*
CHF	0.008	0.018	0.029	0.037	0.045	0.060	0.070	0.078	0.082	0.009	0.002	0.001	-0.001	0.007	0.013	0.005	0.010	0.015
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.002***	*0.589*	*0.899*	*0.899*	*0.455*	*0.263*	*0.621*	*0.409*	*0.303*
GBP	0.009	0.021	0.034	0.035	0.037	0.035	0.030	0.031	0.033	0.010	0.002	-0.007	-0.020	-0.020	-0.029	-0.038	-0.023	-0.024
	0.000	***0.000***	***0.000***	***0.001***	***0.002***	***0.009***	***0.069***	***0.102***	***0.104***	***0.000***	*0.684*	*0.236*	***0.029***	***0.051***	***0.013***	***0.024***	*0.266*	*0.194*
	R-squared									R-squared								
AUD	0.068	0.204	0.304	0.360	0.300	0.250	0.207	0.210	0.209	0.839	0.831	0.824	0.824	0.815	0.823	0.822	0.821	0.819
EUR	0.052	0.082	0.045	0.006	0.001	0.007	0.032	0.064	0.049	0.801	0.773	0.759	0.756	0.737	0.716	0.697	0.677	0.702
JPY	0.099	0.129	0.140	0.120	0.089	0.057	0.033	0.025	0.020	0.837	0.802	0.800	0.800	0.800	0.809	0.806	0.802	0.798
NZD	0.051	0.106	0.108	0.108	0.099	0.093	0.112	0.164	0.209	0.833	0.837	0.827	0.827	0.825	0.824	0.825	0.826	0.826
CHF	0.061	0.117	0.148	0.150	0.170	0.204	0.217	0.221	0.216	0.839	0.829	0.828	0.826	0.826	0.825	0.822	0.821	0.820
GBP	0.066	0.112	0.133	0.096	0.076	0.049	0.027	0.023	0.024	0.833	0.818	0.818	0.823	0.828	0.830	0.830	0.821	0.819
	Out-of-sample Results																	
	Area Under ROC Curve									Area Under ROC Curve								
EUR	0.671	0.773	0.817	0.832	0.807	0.782	0.769	0.774	0.774	0.972	0.968	0.956	0.958	0.949	0.950	0.947	0.949	0.949
JPY	0.661	0.685	0.639	0.554	0.509	0.556	0.604	0.671	0.639	0.965	0.946	0.942	0.952	0.928	0.935	0.928	0.920	0.931
NZD	0.676	0.692	0.714	0.692	0.657	0.616	0.581	0.556	0.541	0.970	0.945	0.946	0.946	0.946	0.951	0.948	0.947	0.946
CHF	0.648	0.707	0.703	0.694	0.690	0.686	0.693	0.733	0.760	0.967	0.969	0.951	0.953	0.950	0.952	0.954	0.958	0.961
GBP	0.640	0.684	0.701	0.700	0.722	0.743	0.748	0.755	0.752	0.971	0.951	0.949	0.952	0.948	0.949	0.946	0.949	0.954
UK	0.652	0.704	0.721	0.691	0.649	0.613	0.589	0.600	0.579	0.967	0.954	0.951	0.956	0.955	0.957	0.960	0.951	0.952

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta \widetilde{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta \widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$ is the relative stock market return, the explanatory variable. $\widetilde{X}_{t-k}$ is the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

Next, we built on the observation that exchange rates are asset prices and should be driven by expectations about future economic conditions. In the business and financial cycles literature, term spread, which embodies agents' expectation about future economic dynamics, has been found to be a robust predictor of cyclical episodes (Estrella and Mishkin, 1997; Chen, 2009; Mishkin, 1991). The results employing relative term spread as an explanatory variable are presented in Table 3.7. The estimates from the simple probit in panel A demonstrate that a rising relative term spread predicts depreciation cycles of the domestic currency. As argued earlier, the term spread may yield ambiguous predictions depending on whether inflation expectations or the reduction in risk premium dominate. In this case, news about inflation expectations is more important for the set of currencies being studied. For the EUR however, the model implies depreciation for upto a year and subsequent appreciation. When the persistence of exchange rate cycles is incorporated via the dynamic probit (panel B), the term spread remains a useful predictor over a quarter for the Japanese yen only. This is in contrast to evidence from the business cycle literature, where the yield curve has been shown to be a consistent predictor using both static and dynamic models (see e.g., Stock and Watson, 1993; Estrella and Mishkin, 1997; Kauppi and Saikkonen, 2008)[8]. The in-sample fit, R^2, of the static model is rather poor compared to the other predictors examined so far. Although the explanatory variable itself is not significant in the dynamic probit model (panel B), the in- and out-of-sample fits are quite high. This could reflect the fact that knowledge of a previous bear or bull state already incorporates future expectations about the path of economy such that the information content of the term spread is less useful for cyclical predictions of exchange rates.

[8]However, we also note that the three Neslon-Seigel factors implied by the yield curve have recently been found by Chen and Tsang (2013) to be significant predictors of changes in the exchange rate. Because the three factors have been found to proxy the long term inflation expectations, business cycles and the central banks' monetary stances (Dewachter and Lyrio, 2006; Rudebusch and Wu, 2008), the significance of individual factors should not be surprising. Furthermore, as Chen and Tsang (2013) use linear models without persistence, the results should roughly be comparable to our static model, which indeed demonstrates the significance of term spread for the prediction of depreciation cycles.

Table 3.7: Probit Model Estimation Results with Relative Term Spread

h	1	3	6	9	12	15	18	21	24	1	3	6	9	12	15	18	21	24
	Panel A: Simple Probit									Panel B: Dynamic Probit								
	In-sample Results																	
	$\widehat{\beta}$									$\widehat{\beta}$								
AUD	-0.014	0.011	0.028	0.084	0.140	0.173	0.197	0.189	0.140	-0.009	0.046	0.050	0.093	0.098	0.102	0.086	0.070	-0.032
	0.703	*0.777*	*0.465*	***0.030***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.892*	*0.455*	*0.398*	*0.161*	*0.178*	*0.113*	*0.167*	*0.266*	*0.629*
EUR	0.505	0.467	0.323	0.275	0.136	-0.103	-0.262	-0.267	-0.244	0.104	0.015	0.000	0.023	-0.107	-0.245	-0.220	-0.106	-0.076
	0.001	***0.000***	***0.004***	***0.004***	*0.139*	*0.290*	***0.007***	***0.009***	***0.017***	*0.552*	*0.939*	*0.998*	*0.889*	*0.467*	*0.161*	*0.195*	*0.591*	*0.657*
JPY	0.319	0.388	0.351	0.272	0.298	0.210	0.106	0.014	-0.056	0.168	0.211	0.084	0.055	0.104	-0.018	-0.068	-0.108	-0.064
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.009***	*0.747*	*0.202*	***0.006***	***0.003***	*0.177*	*0.408*	*0.191*	*0.788*	*0.316*	*0.085*	*0.368*
NZD	0.150	0.140	0.110	0.069	0.057	0.065	0.049	0.053	0.053	0.055	0.047	0.009	-0.041	0.039	0.021	0.001	0.035	0.021
	0.000	***0.000***	***0.000***	***0.019***	***0.042***	***0.018***	*0.074*	***0.049***	***0.049***	*0.316*	*0.452*	*0.845*	*0.420*	*0.352*	*0.709*	*0.978*	*0.261*	*0.530*
CHF	0.202	0.219	0.198	0.195	0.233	0.211	0.180	0.179	0.152	0.141	0.099	0.046	0.094	0.137	0.055	0.063	0.068	0.036
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	*0.061*	*0.191*	*0.509*	*0.165*	*0.055*	*0.470*	*0.342*	*0.290*	*0.587*
GBP	0.197	0.160	0.108	0.122	0.138	0.108	0.102	0.130	0.150	0.058	-0.015	0.005	0.053	0.045	-0.008	0.049	0.080	0.044
	0.000	***0.000***	***0.009***	***0.003***	***0.001***	***0.011***	***0.017***	***0.002***	***0.000***	*0.428*	*0.833*	*0.949*	*0.475*	*0.561*	*0.909*	*0.466*	*0.288*	*0.534*
	R-squared									R-squared								
AUD	0.000	0.000	0.001	0.012	0.034	0.050	0.064	0.059	0.034	0.821	0.821	0.820	0.821	0.820	0.827	0.825	0.823	0.820
EUR	0.146	0.118	0.059	0.040	0.010	0.006	0.041	0.042	0.036	0.781	0.773	0.762	0.750	0.738	0.731	0.712	0.686	0.703
JPY	0.140	0.187	0.159	0.102	0.112	0.061	0.017	0.000	0.005	0.812	0.816	0.803	0.801	0.803	0.808	0.807	0.806	0.800
NZD	0.067	0.060	0.040	0.017	0.012	0.015	0.009	0.010	0.010	0.823	0.822	0.828	0.828	0.826	0.824	0.822	0.822	0.820
CHF	0.061	0.071	0.061	0.058	0.080	0.067	0.050	0.050	0.037	0.828	0.823	0.829	0.830	0.832	0.825	0.824	0.822	0.820
GBP	0.051	0.034	0.016	0.021	0.027	0.017	0.015	0.024	0.031	0.820	0.818	0.817	0.817	0.825	0.823	0.821	0.821	0.817
	Out-of-sample Results																	
	Area Under ROC Curve									Area Under ROC Curve								
AUD	0.532	0.503	0.565	0.548	0.608	0.606	0.625	0.628	0.593	0.951	0.951	0.951	0.952	0.954	0.958	0.955	0.957	0.952
EUR	0.758	0.711	0.648	0.601	0.512	0.484	0.610	0.604	0.631	0.944	0.942	0.940	0.942	0.939	0.946	0.943	0.938	0.926
JPY	0.753	0.779	0.758	0.700	0.685	0.621	0.561	0.505	0.550	0.960	0.964	0.952	0.946	0.947	0.948	0.948	0.949	0.948
NZD	0.624	0.615	0.616	0.582	0.544	0.558	0.546	0.553	0.542	0.948	0.952	0.956	0.953	0.954	0.952	0.950	0.954	0.946
CHF	0.648	0.661	0.649	0.645	0.656	0.638	0.612	0.615	0.588	0.961	0.956	0.957	0.959	0.960	0.955	0.954	0.954	0.949
GBP	0.605	0.576	0.541	0.548	0.566	0.559	0.566	0.582	0.592	0.945	0.951	0.949	0.948	0.953	0.950	0.943	0.953	0.948

Notes: i. Probit models: Simple, $Pr(S_t = 1) = \Phi(\alpha + \beta \widetilde{X}_{t-k})$, and dynamic, $Pr(S_t = 1) = \Phi(\alpha + \beta \widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$ is the relative term spread, the explanatory variable. $\widetilde{X}_{t-k}$ is the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

Lastly, we estimate the two encompassing models, simple and dynamic, containing all five variables that have previously been used as single predictors. Table 3.8 and 3.9 present the results. Under the simple model, unlike the individual significance of previously observed, different combinations of variables are significant for different currencies. For the euro, for example, EROCT, RPPP, TED and term spreads exhibit predictability, while for Australian dollar, these variables are significant at the two quarter time period. No single variable is significant for all the currencies. However, except for the NZ dollar and the GB pound, term spread is a useful predictor in the simple version of encompassing model. When dynamics are added via lagged cycles, four variables, viz., EROCT, RPPP, term spread and equity return differential, provide predictability only for the exchange cycles of the Australian dollar. As with earlier estimation results with single predictor, the only variable that is consistently significant is their own dynamic persistence.

Table 3.8: Estimation Results: Simple Probit - An Encompassing Model

k	1	3	6	1	3	6	1	3	6
	Australia			Euro-area			Japan		
EROCT	0.021	0.062	0.243	-0.018	0.158	0.697	-0.070	-0.115	-0.132
	0.507	*0.291*	***0.002***	*0.649*	***0.039***	***0.000***	***0.008***	***0.047***	*0.054*
RPPP	0.044	0.114	0.314	0.016	0.229	0.821	-0.050	-0.085	-0.096
	0.160	*0.059*	***0.000***	*0.678*	***0.006***	***0.000***	*0.064*	*0.139*	*0.152*
TED	-0.495	-0.690	-0.555	-0.686	-0.074	-0.012	-0.232	-0.231	-0.225
	0.078	***0.012***	*0.119*	***0.044***	*0.875*	*0.981*	*0.194*	*0.236*	*0.277*
SPRD	0.138	0.142	0.367	0.858	0.732	0.743	0.570	0.467	0.275
	0.260	*0.319*	***0.028***	***0.000***	***0.002***	***0.001***	***0.000***	***0.002***	***0.051***
RSER	0.000	0.004	0.014	-0.002	-0.005	-0.016	0.003	0.006	0.010
	0.868	*0.529*	*0.135*	*0.737*	*0.608*	*0.277*	*0.148*	*0.162*	*0.109*
R2	0.257	0.416	0.513	0.479	0.546	0.595	0.487	0.517	0.456
AUC	0.791	0.857	0.894	0.882	0.916	0.919	0.888	0.896	0.867
	New Zealand			Switzerland			United Kingdom		
EROCT	0.003	0.041	0.214	-0.045	-0.040	0.036	-0.027	-0.010	-0.005
	0.949	*0.597*	*0.055*	*0.166*	*0.505*	*0.678*	*0.321*	*0.864*	*0.952*
RPPP	0.020	0.068	0.240	-0.022	0.000	0.075	-0.004	0.035	0.046
	0.621	*0.390*	***0.041***	*0.499*	*0.996*	*0.402*	*0.877*	*0.567*	*0.578*
TED	-0.468	0.127	0.345	-0.819	-0.869	-0.930	-0.793	-1.023	-0.994
	0.371	*0.766*	*0.419*	***0.025***	***0.020***	***0.011***	*0.053*	***0.012***	***0.015***
SPRD	-0.007	-0.120	-0.140	0.261	0.259	0.196	0.128	0.134	0.109
	0.978	*0.671*	*0.609*	***0.013***	***0.012***	*0.103*	*0.260*	*0.271*	*0.402*
RSER	0.009	0.025	0.029	-0.002	0.000	0.013	0.002	0.005	0.005
	0.122	*0.071*	*0.192*	*0.531*	*0.960*	*0.152*	*0.444*	*0.586*	*0.759*
R2	0.350	0.469	0.380	0.378	0.447	0.363	0.243	0.321	0.289
AUC	0.855	0.894	0.847	0.837	0.867	0.849	0.772	0.818	0.811

Notes: i. Model - $Pr(S_t = 1) = \Phi(\alpha + \sum_{k=0}^{q} \beta \widetilde{X}_{t-k})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$, the set of explanatory variables, are the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

Table 3.9: ESTIMATION RESULTS: DYNAMIC PROBIT - AN ENCOMPASSING MODEL

k	1	3	6	1	3	6	1	3	6
	AUSTRALIA			EURO-AREA			JAPAN		
EROCT	0.017	0.134	0.207	-0.022	0.191	0.565	-0.007	0.041	-0.019
	0.612	***0.012***	***0.002***	*0.569*	***0.017***	***0.000***	*0.838*	*0.354*	*0.751*
RPPP	0.044	0.139	0.209	0.017	0.212	0.616	0.019	0.042	-0.033
	0.210	***0.012***	***0.004***	*0.655*	***0.014***	***0.000***	*0.549*	*0.339*	*0.570*
TED	-0.113	-0.532	-0.258	0.308	0.197	0.229	-0.182	-0.198	-0.165
	0.712	*0.179*	*0.440*	*0.377*	*0.541*	*0.554*	*0.335*	*0.329*	*0.365*
SPRD	0.045	0.282	0.322	0.004	0.066	0.175	0.299	0.322	0.099
	0.665	***0.014***	***0.011***	*0.982*	*0.704*	*0.230*	*0.038*	***0.006***	*0.444*
RSER	0.002	0.014	0.015	-0.005	0.002	-0.011	0.005	0.004	0.005
	0.262	***0.037***	***0.060***	*0.392*	*0.758*	*0.292*	*0.149*	*0.325*	*0.369*
S_{t-1}	3.516	3.043	3.101	3.563	3.091	2.982	3.439	2.969	3.258
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***
R2	0.871	0.830	0.837	0.867	0.804	0.818	0.883	0.822	0.811
AUC	0.986	0.972	0.969	0.982	0.964	0.971	0.980	0.964	0.959
	NEW ZEALAND			SWITZERLAND			UNITED KINGDOM		
EROCT	0.074	0.008	0.204	0.056	0.116	0.124	-0.009	0.052	-0.190
	0.199	*0.924*	***0.074***	*0.150*	*0.070*	*0.133*	*0.836*	*0.253*	*0.092*
RPPP	0.098	0.021	0.194	0.088	0.125	0.090	0.036	0.060	-0.193
	0.102	*0.785*	*0.104*	***0.025***	*0.057*	*0.305*	*0.354*	*0.203*	*0.077*
TED	0.949	0.185	0.643	-1.177	-0.879	-0.419	-1.172	-0.536	-0.344
	0.109	*0.624*	*0.215*	***0.014***	***0.028***	*0.219*	***0.008***	*0.152*	*0.463*
SPRD	-0.403	-0.132	-0.334	0.172	0.164	0.135	0.004	0.004	-0.341
	0.278	*0.617*	*0.156*	*0.060*	*0.093*	*0.165*	*0.976*	*0.967*	*0.058*
RSER	0.006	0.021	0.011	0.004	-0.006	0.007	0.000	-0.002	-0.011
	0.532	*0.240*	*0.569*	*0.347*	*0.336*	*0.445*	*0.943*	*0.819*	*0.434*
S_{t-1}	3.440	2.076	3.056	3.926	3.214	4.129	3.990	3.088	3.130
	0.000	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***	***0.000***
R2	0.812	0.727	0.742	0.902	0.835	0.842	0.879	0.795	0.763
AUC	0.971	0.959	0.959	0.988	0.964	0.966	0.983	0.946	0.944

Notes: i. Model - $Pr(S_t = 1) = \Phi(\alpha + \sum_{k=0}^{q} \beta \widetilde{X}_{t-k} + \gamma S_{t-1})$, where S_t is cycle dummy and $\widetilde{X}_{t-k}$, the set of explanatory variables, are the relative difference between the domestic and the foreign variable. ii. For in-sample results, parameter estimates for $\widehat{\beta}$ and Pseudo-R^2 proposed in Estrella (1998) are reported. iii. Numbers in *Italics* are the robust $p-$values. iv. Bold entries show significance at 5% or below. v. For out-of-sample results, the area under receiver-operating-characteristic (ROC) curve is reported. The range of AUC is [0.5-1), with higher values implying a better model. v. Sample period: 1980M1-2012M12.

3.5 Conclusion

Previous research has attempted to predict either the levels or the changes in the bilateral exchange rates. In the present study, we build on Mussa (1976, 1979) and view exchange rates as asset prices. Therefore, like the equity prices, the exchange rate should be forward looking and should be driven by investor expectations about economic conditions. Furthermore, the cyclical behavior (bulls and bears) of equity prices is well documented. However, despite being inherently cyclical, no attempts have been made to study the nature and the driving forces of these cyclical episodes. Our study, therefore, takes a different course; we attempt to forecast the direction of change in exchange rates by exploiting a set of variables that embody market expectations. Specifically, we focus separately on deviations from two of the widely respected parity conditions viz., the uncovered interest rate parity (UIP) and the relative purchasing power parity (RPPP), and consider the equity market return differentials, the TED spread and the yield spread. The deviations and the spreads represent broad risk factors in the foreign exchange market. Furthermore, by employing these variables, we also aim to illuminate commonly observed violations of UIP and RPPP conditions. Violation of UIP is particularly interesting to currency traders. Our empirical tools are two versions of a non-linear binary choice probit model. We first employ a static (simple) specification and augment it with a lagged dependent variable based on the observation that cycles tend to persist. By adding the lagged dependent variable, we essentially increase our information set.

Our results broadly show that violations of UIP are exploited by currency traders who earn abnormal returns by engaging in carry trades. When unspotted (static probit), the deviations from UIP may persist over a year. However, once spotted and market actors integrate that information (dynamic probit), the violations become a short run phenomenon. Indeed, our dynamic model predicts a change of course of exchange rate (e.g., appreciation to depreciation) for the euro (at the 12 month horizon), the Swiss franc (at 6-12 months) and the GB pound (at 9-12 months). These reversals and the corresponding unwinding of carry trade positions can induce liquidity pressures in the market and lead to liquidity spirals as predicted in Brunnermeier and Pedersen (2009). In fact, an increase in the TED spread, which we employ to measure funding pressures in the market also predicts an appreciation of funding currencies and a corresponding depreciation of investment currencies in the dynamic probit model, implying an amplification of losses on carry trades. Interestingly, deviations from RPPP are also of a short-term nature under the dynamic model. The results under a pseudo-parity condition where we replace the commodity price index with the equity market index imply portfolio switching between the two stock markets and resulting depreciation of the home currency. However, these switches are short run when accounting for persistence. Based on the comparison of results from the static and dynamic probit models, we believe that the short-term nature of violations of the UIP, the RPPP and the pseudo-parity with the equity price index reflect agents' revision of their beliefs in light of information updates. Consistent with the fact that the countries under investigation implicitly or explicitly target inflation, any relative increase in inflation expectations leads to a depreciation of the domestic currency. The dynamic probit predicts depreciation due to inflation expectations upto a quarter and a subsequent reversal. This pattern is consistent with the theoretical and empirical findings in the literature, e.g., overshooting. Finally, the relative term spread is found to predict depreciation cycles (bears) only in the static version of probit model but not utilizing the dynamic specification.

The results suggest that our simple framework can be employed to predict the cyclical episodes of exchange rate movement. The behavior and nature of cyclical variations explained by the violations of the UIP, the RPPP and the liquidity risk measure have important implications for investors as well as policy makers. For example, in a world of increasing capital mobility, portfolio investment has gained importance for both investors and governments. While an appreciating US dollar, for instance, might be beneficial to a foreign investor, e.g., a hedge or a sovereign wealth fund holding a portfolio of the US financial assets, a reversal might corrode its accumulated advantage. For the US, however, such an appreciation erodes the international competitiveness of US produced goods and services and might prompt corrective action by the Fed. For investors, therefore, model based indications of probable depreciation might help investors diversify their investments away from the US. For the most active class of forex investors, i.e., the carry traders, signals of depreciating funding currencies and subsequent appreciation can help avoid losses on carry trades via portfolio re-balancing. For policy makers, as the level of exchange rate determines the international competitiveness, the depreciation/appreciation signals obtained from the model can help ensure timely corrective measures to smooth dysfunctions or sustained currency misalignment. Moreover, empirically established that the exchange rates indeed follow periods of upswings and downswings, and suggest that early warnings signals transmitted by models can help both investors and policy makers take measures to avoid carry trade induced liquidity spirals.

3.6 Appendix

Data Sources

Table 3.10: Data Sources

S. No.	Series	Period	Source
1.	Money Market Rate	1980:M1 to 2012:M12	IMF - IFS
2.	Treasury Bills Rate (3M)	1980:M1 to 2012:M12	IMF - IFS
3.	Long Term Bond Rates	1980:M1 to 2012:M12	IMF - IFS
4.	LIBOR	1986:M2 to 2012:M12	FRED
5.	Exchange Rates	1980:M1 to 2012:M12	FRED
6.	Consumer Price Index	1980:M1 to 2012:M12	OECD
7.	Industrial Production Index	1980:M1 to 2012:M12	OECD
8.	Stock Market Index	1980:M1 to 2012:M12	OECD
	(Monthly averages, 2005=100)		(via Haver DLX)
	- Australia (ASX Ordinaries)		
	- Euro Area (DJ Euro STOXX Broad)		
	- Japan (TOPIX Index)		
	- New Zealand (NZSE Capital Index)		
	- Switzerland (SBC 100 Index)		
	- USA (NYSE Commons)		
	- UK (FTSE)		

Notes: i) All the data for Euro-area is from 1999:M1 to 2012:M12. ii) For New Zealand, due to unavailability of treasury bill rates, the 90-day interbank rate is used. The rates, available from 1985:M1, were extracted from FRB New Zealand. iii) Where applicable, the series are seasonally adjusted.

Chapter 4

Business and Financial Cycle Synchronization[1]

4.1 Introduction

A reliable assessment of how synchronized different countries' business cycles are is of potential importance for policymakers: it implies they should better coordinate their monetary and fiscal policies for sake of increasing policy effectiveness (Mundell, 1961). Moreover, the current eurozone turmoil has painfully reminded us that a certain degree of cross-country business cycle synchronization is important for monetary unions to be viable in the longer-term. In other words, one may argue that the current degree of eurozone economic integration - as reflected by business cycle synchronization proxies - is too low for a viable currency union[2].

There is no academic consensus on how to measure business cycle synchronization or what the critical lower bound should be below which monetary

[1]This chapter is based on 'Business and Financial Cycle Synchronization' co-authored with Sajid M Choudhry and Stefan Straetmans.

[2]Krugman and Obstfeld (2009) mentions four conditions to be fulfilled for a successful monetary union: strong intra-regional trade flows, sufficient labor mobility, similar economic structure and fiscal federalism. Since the introduction of the single currency, there has been an increase in intra-regional trade but other currency union requirements have been lagging behind. For example, labor has been largely immobile because of, e.g., legal and cultural reasons, social costs of migration etc. The eurozone crisis has also revealed structural differences between Northern and Southern countries in the eurozone. Breuss (2011) points out persistent weaknesses in competitiveness (due to, e.g., diverging cross-country unit labor costs) of some peripheral member states. As far as fiscal federalism is concerned, the eurozone does not have authority to levy taxes. Moreover, there is hardly any public support for an expansion of the current EU budget. Thus, eurozone money transfers between the nation states are relatively limited due to the limited scale of the eurozone budget. Also, the governments that have bailed out their institutions have put their home country taxpayer's interest first.

unions are non-sustainable. The focus of this chapter is not on the latter issue but tries to contribute to the *measurement* issue of eurozone business cycle synchronization. As a benchmark for comparison, we also study the degree of financial cycle synchronization for the same eurozone country pairs. Both measures of integration should presumably have been affected by the introduction of the single currency and it is therefore interesting to investigate both concepts in parallel. Moreover, real and financial market integration are expected to be interrelated because (i) financial market fluctuations reflect the markets' expectations about future real economic activity and because (ii) cross-border trade flows require cross-border financial flows (albeit a large fraction of financial flows is admittedly not trade-related nowadays). However, we limit ourselves to making up a state of the current real and financial eurozone linkages by looking at cyclical co-movements in real and financial series separately. The issue of financial synchronization is also economically relevant from an investment perspective. For example, the study of financial cycles and their co-movements across assets or across borders can be a valuable tool for investors who want to rebalance their portfolios (buy and sell signals can be obtained by means of the troughs and peaks of the identified cycles; as such a framework that determines bulls and bears as well as their co-movements can be an additional toolkit for tactical asset allocation). Finally, persistent cross-border swings in financial markets can be potentially destabilizing for the eurozone and in the end also for the real economy which suggests that a proper monitoring of (increases in) financial synchronization by regulatory bodies and policymakers is desirable. Candelon et al. (2008b) provide a more elaborate discussion of potential implications.

Measuring the degree of cyclical synchronization between economic or financial variables necessarily requires two steps because it demands the determination of cycle phases prior to estimating the degree of cyclical synchronization. How to determine the cycle phases in itself will not be the main focus in this chapter. We rather focus on the synchronization measurement issue using the periods of sustained rises and falls reflected by the dummy phase variable as an input. We will opt for the Bry and Boschan (1971) nonparametric dating algorithm that maps original time series on binary 0/1 series that either reflect "bull and bear" periods (financial data) or "expansion and recession" periods (real output data).

A growing body of literature already exists on cyclical synchronization, mainly for real time series (especially business cycles) and to a lesser extent also for financial markets (the co-movement of stock market cycles). Existing studies on real and financial globalization often reach mixed conclusions on both the degree of real and financial integration/synchronization as well as on the driving forces behind it. Nowadays, because of globalization the popular assumption is that increased cross-border flows in real goods (international trade flows) and financial assets (financial capital flows) have led to an increase in both real and financial synchronization. Indeed, a lot of developing economies have put themselves on the path towards real and financial liberalization, often incentivized by large IMF support packages. As for Europe, the creation of the single market (in 1992) and the introduction of the single currency (in 1999) was also aimed at boosting intra-European trade and financial globalization. This is supposed to have led to stronger synchronization of real and financial cycles in the eurozone - an important precondition for the currency area to deliver optimal benefits to the participants of the currency union. However, the academic consensus on both presence and causes of real and financial globalization is not as strong as one would expect.

For business cycles, for example, there are both studies which support the idea of increased synchronization (Artis and Zhang, 1997, 1999; Artis et al., 2004; Imbs, 2004; Kose et al., 2008) but others argue that there has been a global de-synchronization or regional specialization (Krugman, 1991; Krugman and Venables, 1993; Kalemli-Ozcan et al., 2001). The latter papers argue that regions with a more specialized production structure exhibit output fluctuations that are less correlated with those of other regions with less symmetric fluctuations. Doyle and Faust (2002) and Doyle and Faust (2005) were also unable to find clear evidence of an increase in correlation of growth rates of output, consumption, or investment.

A variety of studies focused on the degree of European business cycle synchronization (De Haan et al., 2008). The picture that emerges from this literature survey is that European business cycles have gone through periods of convergence as well as divergence but that the synchronization has increased during the 1990s. The empirical evidence with regard to European financial market synchronization is also mixed. While Corhay et al. (1993), Taylor and Tonks (1989), Knif and Pynnonen (1999) and Dickinson (2000) provide support for increased financial integration, Chan et al. (1997) and Gerrits and Yuce (1999) find opposite results. Upon analyzing European equity market integration since the 1980s, Fratzscher (2002) finds that European equity markets have become more strongly integrated after 1996 (which is confirmed by Kim et al., 2005). Harding and Pagan (2006) find relatively weak evidence of business cycle synchronization but stronger evidence of financial cycle synchronization.

Notice that the measurement frameworks of the vast majority of cited studies are essentially *linear* in nature. However, non-linear behavior may play an important role in determining business and financial cycle linkages. More specifically, there seems to be consensus in the academic literature as well as amongst practitioners that phenomena like systemic financial crises and contagion spillovers are essentially non-linear in nature which may also explain why traditional (linear) models perform very poorly in predicting these crisis events. As concerns the importance of non-linearities for studying business cycle synchronization, Helbling and Bayoumi (2003) argue that joint slowdowns in economic activity actually represent structural breaks or asymmetries and are thus fundamentally non-linear in nature. This chapter's main contribution lies in approaching the synchronization of business and financial cycles in a non-linear fashion by means of a simple probit model. Both the dependent as well as independent variables are binary series depicting business or financial cycles. This approach allows us to see the economic impact of cyclical relations in a probabilistic way by calculating the marginal effects. Moreover, unlike linear correlations, the model can be used for predicting the level used for calculating and predicting the level of cycle synchronization. To our knowledge,the use of simple binary response models like probit or logit regressions is novel to the empirical literature on real and financial synchronization.

We argued above that there is no strong consensus in the - mainly linear - empirical synchronization literature on whether business and financial cycle synchronization has actually increased or dropped after the introduction of the euro. We wonder whether the contradictory results may be due to neglecting non-linear components in real and financial linkages and we therefore apply probit models to European real and financial cycle data. We focus on bilateral real and financial linkages for 9 eurozone economies for the period from January 1960 to December 2010. We first estimate a simple probit model where both left and right hand variables are monthly binary series. However, as we are focusing on the eurozone, the problem of endogeneity might also arise (Frankel and Rose, 1998). We tackle this issue by estimating the probit model via generalized method of moments (GMM), using the lagged independent variable as an instrument. To ascertain whether non-linear models are worth going for as compared to the pure linear approach, we rank the countries based on the marginal probit effects and contrast them with linear correlation ranks.

Anticipating our results, we find that real and financial linkages and their corresponding marginal effects in a probit framework are mostly statistically and economically significant. However, ranks of country pairs do not differ much depending on whether the ranks are based on linear synchronization measures like correlation or more general synchronization measures based on probit frameworks[3]. Moreover, probit regression outcomes suggest a stronger financial than business cycle synchronization in the eurozone. The high degree of eurozone financial synchronization provides an empirical justification for the foreseen creation of a European Banking Union. The introduction of the euro also seems to have influenced the level of business cycle synchronization over time: a few countries' business cycles de-couple from each other over the post-1999 sample but the majority of countries' business cycles gets actually more strongly synchronized. For the full sample, GMM estimates do not strongly alter the estimates of the baseline simple probit model. However, when augmented with an interaction term between cycle variables and the single currency dummy, the GMM estimation alters estimates. We believe the increased synchronization in eurozone business cycles since 1999 is in large part due to the common monetary policy pursued by the European Central Bank. More specifically, common monetary policy acts as a "common factor" in the aggregate demand of the different eurozone countries.

The rest of the chapter is structured as follows. Section 4.2 briefly explains the methodology used; data and empirical results are discussed in section 4.3 while section 4.4 concludes by outlining the consequences of our findings for investors and policymakers.

4.2 Methodological Framework

In this section we briefly outline the econometric methodology to identify cyclical linkages in business cycles and financial cycles. Evidently, prior to measuring cyclical synchronization, one needs to determine the cycles itself. One approach could be to use the National Bureau of Economic Research (NBER) data for business cycles in US real GDP whereas the Economic Cycle Research

[3] This does not imply that non-linearities do not exist.

Institute (ECRI) also publishes business cycle dummy variables for a number of European countries. However, we will not make use of these publicly available cycle data because the ECRI database only contains part of the eurozone countries we are interested in. Moreover, we also focus on financial cycles, which are not stored in publicly available databases of renowned research institutes.

As we do not use predetermined cycle data, the question arises how to determine the real and financial cycles ourselves. Let $y_{i,t}$ denote the (log) stock price or the output series for a certain country i at time t $(i = 1, \cdots, n; t = 1, \cdots, T)$. (Financial) bulls and bears or (real) recessions and expansions are determined using the marginal transform $\varphi(\cdot)$ such that $\varphi(y_{i,t}) = S_{it}$ (for all i) where S_{it} is 1 or 0 in case of a bear (recession) or bull (expansion) period, respectively. There are two main methodological strands in the literature to select $\varphi(\cdot)$. First, Hamilton (1989) imposes a two regime Markov-switching model on $\varphi(\cdot)$ that allows for persistent upward and downward swings in $y_{i,t}$[4]. We prefer, however, a nonparametric approach which can be motivated by the complex temporal behavior of both real and financial time series and the resulting risk of model misspecification. More specifically, we opt for a popular (nonparametric) dating algorithm (Bry and Boschan, 1971)[5]. Applications of this algorithm to financial market data can found in, e.g., Edwards et al. (2003a), Pagan and Sossounov (2003), Candelon et al. (2009) and Chen (2009). The algorithm recognizes time series patterns, detaches these patterns according to a sequence of rules and then locates the turning points (peaks and troughs) in the series. The employed rules, however, are typically not taken to be identical for business and financial cycle determination. Pagan and Sossounov (2003) observe that the nature of financial asset prices is sufficiently different from real quantities so that the algorithm should be implemented in slightly different ways[6]. The location of turning points amounts to identifying local maxima or minima within a window of k months. More specifically, a turning point represents a peak at time t if $y_{t-k}, ..., y_{t-1} < y_t > y_{t+1}, ..., y_{t+k}$

[4]See also Hamilton and Lin (1996) and Maheu and McCurdy (2000).

[5]Alternative non-parametric filters for extracting cycles that have been proposed in the literature include, inter alia, the Hodrick and Prescott (1997) filter, the band-pass filter of Baxter and King (1999) and the procedure due to Christiano and Fitzgerald (2003).

[6]Based on reviewing the literature, we implement 4 censoring criteria. First, we set a window length of six months for business cycles and eight months for financial markets. Second, we assume that a complete business cycle and a complete financial cycle cannot take longer than 15 and 16 months, respectively. Third, we impose phase durations of 6 months and 4 months for business and financial cycles, respectively. Fourth, peaks and troughs have to alternate. Finally, we choose the highest of the peaks and the lowest of the troughs in case of multiple peaks or troughs.

whereas it represents a trough if $y_{t-k}, ..., y_{t-1} > y_t < y_{t+1}, ..., y_{t+k}$. Finally, periods from peak to trough are classified as *bears* or *recessions* $(S_t = 1)$ while those from trough to peak are classified as *bulls* or *expansions* $(S_t = 0)$ [7].

After introducing an algorithm towards identifying time series cycles in the form of 0/1 variables, we are ready to perform co-movement analysis. The approach most commonly implemented towards measuring cyclical co-movement that consists of performing linear correlation analysis on business or financial cycles, see, e.g., Harding and Pagan (2006) or Candelon et al. (2009) on business cycle synchronization or Candelon et al. (2008b) on financial synchronization. Although our probit approach is able to pick up both linear and non-linear synchronization, we nevertheless perform some correlation analysis as a benchmark for comparison. A probit basically enables one to calculate the probability that one country's economy gets into recession (or, alternatively, one country's stock market gets bearish) given the state of another country's economy or stock market. The relationship between the cycles of country i and j in its simplest form can be expressed as:

$$y^*_{i,t} = \alpha + \beta S_{j,t} + \varepsilon_t, \tag{4.1}$$

where $y^*_{i,t}$ is an unobservable variable that determines the occurrence of a recession (or, alternatively, stock market bear) at time t and $S_{j,t}$ are the time t business or financial cycle dummies of country j determined with the Bry-Boschan algorithm. However, given that we investigate the co-movement of cycles within the eurozone, this may entail possible endogeneity problems and resulting biases.

Since our interest lies with real and financial (cycle) linkages between eurozone countries, we would also like to know whether the euro introduction has had any impact on the degree of synchronization. To that aim, we define EUD as a euro dummy which is zero before the euro introduction and 1 afterwards. Next, we include an interaction term ($S_{j,t} \times EUD$) in relation (4.1):

$$y^*_{i,t} = \alpha + \beta S_{j,t} + \gamma[S_{j,t} \times EUD] + \varepsilon_t, \tag{4.2}$$

What to expect in terms of single currency impact on business and financial synchronization? First, the introduction of the euro may have enhanced

[7]Notice that this classic cycle definition based on peaks and troughs only makes use of the historical data of a *single* uni-variate time series. It follows that one only needs single country information to determine the cycles in that country variable.

intra-industry trade across the eurozone and this in turn may have increased business cycle synchronization. Using alternative measures of synchronization than the ones we use here, Böwer and Guillemineau (2006) indeed established a positive impact of intra-industry trade on business cycle synchronization. Moreover, we believe that the creation of the European Central Bank and the introduction of a common monetary policy may be even more important for eurozone synchronization, also at the financial side. The ECB policies may effectively act as some underlying common factor in real and financial cycle co-movements.

Since $y^*_{i,t}$ is unobserved, we apply the following censoring rule to get $S_{i,t}$, the corresponding business or financial cycle variable for country i:[8].

$$S_{i,t} = \begin{cases} 1, & \text{if} \quad y^*_{i,t} > 0 \quad \text{and} \\ 0, & \text{if} \quad \text{otherwise,} \end{cases}$$

and where the unobserved variable $y^*_{i,t}$ is equal to the right hand side of equation (4.1)or (4.2). Assuming normality of the error terms (ε_t) in above relations results in a probit model, i.e.,

$$Pr(S_{i,t} = 1) = \Phi(\boldsymbol{X_t}\boldsymbol{\Theta}), \tag{4.3}$$

where $\boldsymbol{X_t} = [1 \quad S_{j,t} \quad (S_{j,t} \times EUD)]$, $\boldsymbol{\Theta} = (\alpha, \beta, \gamma)$ and $\Phi(.)$ is the standard normal distribution[9]. The corresponding log-likelihood is

$$\mathcal{L} = \sum_{t=1}^{T} \left(S_{i,t} \log \Phi(\boldsymbol{X_t}\boldsymbol{\Theta}) + (1 - S_{i,t}) \log[1 - \Phi(\boldsymbol{X_t}\boldsymbol{\Theta})] \right). \tag{4.4}$$

The estimation of parameters, $\boldsymbol{\Theta}$, is carried out via the method of maximum likelihood for the benchmark simple probit model. However, to deal with the possible endogeneity issue due to a single currency area, we exploit the first order conditions (FOC) of (4.4) by noting that

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\Theta}} = \underbrace{\frac{[S_{i,t} - \Phi(\boldsymbol{X_t}\boldsymbol{\Theta})]\phi(\boldsymbol{X_t}\boldsymbol{\Theta})}{\Phi(\boldsymbol{X_t}\boldsymbol{\Theta})[1 - \Phi(\boldsymbol{X_t}\boldsymbol{\Theta})]}}_{A} \boldsymbol{X_t} = 0. \tag{4.5}$$

[8]This is more of a technical artifact. The underlying cycles are either provided by standard sources, e.g., NBER, ECRI or extracted from some reference series by a dating algorithm

[9]Assuming a logistic distribution - and thus modeling the binary linkages with the use of a logit regression - did not dramatically alter our outcomes. The outcomes remain qualitatively the same.

The part labeled 'A' in (4.5) are the '*generalized residuals*' (Gourieroux et al., 1987) and can be exploited to form valid moment conditions for the parameter estimation under 'generalized method of moments (GMM)'.

Finally, since the Θ vector in the probit model is hard to interpret, we also report the marginal effects (*MFX*) of the estimates. Formally, let $\mathbf{y} = (S_{i,t})$. Differentiating (4.3) with respect to $\boldsymbol{X_t}$, we obtain:

$$\frac{\partial E(\mathbf{y}|\boldsymbol{X_t})}{\partial \boldsymbol{X_t}} = \phi(\boldsymbol{X_t}\Theta)\Theta, \tag{4.6}$$

where $\phi(.)$ is the standard normal density. The standard errors for the estimates ($\widehat{\Theta}$) and *MFX* are calculated using the robust covariance as suggested in Greene (2008). Furthermore, marginal effects for continuous variables are calculated at mean values. For the binary (dummy) variables, the MFX are computed as the change in probability when the dummy variable changes from 0 to 1.

Finally, and in contrast to the symmetry restriction $\widehat{\rho}_{ij} = \widehat{\rho}_{ji}$ for classic correlation analysis, a similar property does not hold for estimates of the coefficient vector Θ and corresponding marginal effects, i.e., $\widehat{\Theta}_{ij} \neq \widehat{\Theta}_{ji}$ [10]. Thus, it matters which cycles are selected as dependent and independent variables. However, as our empirical results will show, differences $\left(\widehat{\Theta}_{ij} - \widehat{\Theta}_{ji}\right)$ are typically small and our general conclusions on synchronization remain robust for turning around the LHS and RHS cyclical variable in the probit regression.

4.3 Empirical Results

4.3.1 Data Sources and Description

We download nominal industrial production series and equity price indices from the OECD database and the IFS database, respectively. We managed to gather these data for nine (9) eurozone countries on a monthly frequency between January 1960 and December 2010. The considered countries are: Austria, Belgium, Finland, France, Germany, Greece, Italy, Portugal and Spain. As a consequence, there are $C_9^2 \equiv 36$ bilateral business cycle synchronization pairs and the same number of bilateral financial cycle synchronization pairs to be considered. Moreover, we consider (linear) correlation analysis and (more

[10] In general, regressing a dependent variable y on an independent variable x renders different estimates than regressing x on y and this irrespective of the type of regression (linear/non-linear) considered.

general) probit analysis estimates as well as subsample estimates for the bilateral pairs[11]. Therefore, and in order to keep the dimension of the reported tables manageable, we only report the bilateral results of the bigger European countries (France, Germany, Spain, Italy) with respect to each other and with respect to the smaller countries.

4.3.2 Estimation Results

In this section we would like to address several questions. First, we know from the existing literature that business and financial cycles tend to co-move but the majority of preceding studies used approaches assuming that linkages are basically linear in nature. However, why would real or financial linkages be always linear in nature? In fact, it is increasingly argued in the international macro literature that the nature of cross-country dependence (either between business cycles, financial markets etc.) or crisis spillover phenomena like contagion can probably not be fully captured by linear approaches only. We therefore propose a more general approach towards identifying cyclical linkages. The two forms of co-movements or linkages are obviously not mutually exclusive. The question arises whether real and financial cycles actually exhibit statistically and economically significant synchronization within the probit framework and whether the rankings of country or market pairs are comparable across linear and probit approaches. If rankings differ a lot, this may indicate that the data exhibit non-linear co-movement that could by construction not be detected by linear correlation analysis[12]. Another issue concerns the temporal stability of cross-country linkages over time, i.e., do real economies as well as financial markets get more involved (integrated) through time or do we, on the contrary, observe phenomena like " de-coupling" for some country pairs? We also try to answer that question, both using the traditional linear synchronization measures as well as the probit framework. Moreover, within the context of our data set (a subset of eurozone countries), it is interesting to investigate whether the introduction of the euro represents a structural break in the linear

[11] For sake of convenience, we leave "mixed" real-financial linkages leave this for future research.

[12] This is a sufficient (but not necessary) condition for the existence of non-linear linkages. If we find that probit marginal effects lead to the same ranking of country pairs in terms of synchronization, this may either imply that cycles are only linearly related or that non-linear linkages do not alter the rankings established using linear synchronization measures. The only thing we know for sure is that non-linearities, if present in the data, will only be detected by synchronization indicators that allow for non-linear behavior. Linear correlation analysis does not satisfy this criterion but probit modeling does.

and probit synchronization measures, i.e., whether the euro introduction has actually led to more convergence or, on the contrary, has induced de-coupling and divergence (and if so, between which countries?).

Table 4.1: Business Cycle Dating

Year	Austria		Belgium		Finland		France		Germany		Greece		Italy		Portugal		Spain	
Stylized facts																		
# Cont.	10		12		13		11		12		11		12		14		10	
ADC	14		14		12		15		14		16		19		11		13	
ADE	43		37		33		40		34		38		29		32		42	
Peak and trough dates																		
	P	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**
60-65	60-12	61-8		61-1	65-6				62-8	63-2			64-1	64-8	61-3	62-4		
	65-1	65-12																
66-70	66-10	67-8	67-2	68-7		66-1	66-11	68-5	66-3	67-5	66-5	67-7			66-4	67-2		
					66-11	67-6									70-6			
71-75	74-6	75-10	74-6	75-4	74-7		74-8	75-5	73-8	75-7			71-1	72-3		71-2	74-8	75-4
													74-6	75-4	74-3	75-8		
76-80	79-12		76-10	77-9		77-3	79-8	80-11	79-12	80-9	76-11	78-1	77-1	77-11			77-4	77-11
			79-12		80-7								78-12	79-7			80-3	
													80-3					
81-85		81-6		82-3		81-6	81-12	82-8	81-10	82-11	81-8	83-5		83-5	83-6	84-4		82-8
	82-1	82-12			85-5				83-12	84-6			84-8		84-11	85-9	84-8	85-6
	83-7	84-11													90-8			
86-90	90-12		90-3			86-3	86-4	87-1			86-4	87-6		87-1			90-6	
					87-3	87-11	90-4				90-2	90-9	89-12					
					89-7													
91-95		92-12		91-7		91-6		93-6	91-1	93-7	91-9	93-7		93-7		91-4		91-3
			92-2	94-7	95-1	95-9			94-12	95-10	94-9	95-7	95-12		92-2	92-8	91-12	93-4
			95-5												94-9	95-3	94-12	
96-00	00-12			96-2	98-5	98-12	98-5	99-8	98-7	00-1	96-6	97-8		96-12	96-6	96-12		96-1
			98-7	99-2	00-10		00-12				98-3	99-4	97-12	98-12				
			00-12								00-12		00-12					
01-05		01-11		01-10		01-7		03-5	01-2	01-11		01-12		03-5	02-4	03-3	00-11	01-12
			04-7	05-3	04-12	05-6	05-1	05-10	02-8	03-9					03-9	05-5		
06-10	08-4	09-5	08-4	09-6	06-7	07-1	08-4	09-4	08-2	09-4	08-4		07-8	09-3	06-12	09-1	07-6	09-3
				08-1	09-5	10-5									10-3		10-4	

The abbreviations denote: # Cont. = No. of contractions; ADC = Average duration of contractions; ADE = Average duration of expansion; P = Peak date; T = Trough date.

Table 4.2: Financial Cycle Dating

Year	Austria		Belgium		Finland		France		Germany		Greece		Italy		Portugal		Spain	
Stylized facts																		
# Cont.	12		15		10		13		13		6		12		6		13	
ADC	29		18		24		16		18		22		26		20		18	
ADE	20		26		34		31		27		30		25		23		28	
Peak and trough dates																		
	P	T	P	T	P	T	P	T	P	T	P	T	P	T	P	T	P	T
60-65	62-2		60-12	61-5	61-2	62-1	62-4	64-6	60-9	62-10			60-9	65-1			63-4	64-6
			62-11	64-2	64-3				64-9									
66-70		67-6	67-1	69-5		68-2	66-1	67-7		66-11			66-3	67-3			70-2	70-12
	67-12	69-7	70-7				70-1		69-11				67-10	68-11				
	70-9												69-11					
71-75		71-12		73-6	73-10			71-10		71-10				72-3			74-4	
	73-7	74-10	74-12	75-5			73-5	74-9	73-3	74-9			73-6					
76-80	76-4	79-2	76-11	78-5		77-10	76-2	77-4	76-3	76-10				77-12				78-2
	80-3		78-12	80-2	80-4	80-12	80-10		78-9	80-3							78-7	80-5
81-85		82-11	81-6		84-8	85-7		82-3	81-7	81-12			81-5	82-7			81-8	82-9
	83-5	84-8																
86-90	86-1	88-2		87-8	89-4		87-3	88-1	86-4	88-1	87-10	89-3	86-5	88-2	88-10	89-10	87-8	89-2
	90-3		87-12	90-1			90-5		90-7		90-7		90-6				89-9	90-9
91-95		93-1	91-1	92-5		92-9		91-1		92-9		92-11		92-9	92-11	94-3	91-5	92-9
	94-1	95-11	92-9	94-5	94-2	95-3	92-5	92-10	94-4	95-3	94-1	95-3	94-5	95-12	92-5	92-10	94-1	95-3
			95-3				94-2	95-3										
96-00	98-5			98-7	00-3		00-9		00-2		99-9		00-3			98-4	98-7	99-7
															98-10	00-3	00-2	
01-05		02-10	03-3			03-3		03-3		03-3		03-3		03-3	02-10			02-9
					04-3	04-8			04-1	04-8								
06-10	07-6	09-3		07-5	07-10	09-3	07-5	09-3	07-10	09-2	07-10	09-2	07-5	09-3		07-7	07-10	09-2
			09-3	10-4			10-4				09-10		09-10		09-3	09-10	09-12	

The abbreviations denote: # Cont. = No. of contractions; ADC = Average duration of contractions; ADE = Average duration of expansion; P = Peak date; T = Trough date

Our empirical analysis starts by applying the Bry-Boschan dating algorithm to our countries' industrial production and share prices index time series. This renders turning points and resulting peak and trough phases per country. Notice the dating algorithm is applied on the series in levels (seasonally adjusted but no de-trending.) Tables 4.1 and 4.2 report output of the dating algorithm for the countries' business cycles and financial cycles, respectively. Each of the table's upper panel reports some basic descriptive cycle statistics like the number of economic and financial contractions, and the average duration of expansions and contractions; the tables' lower panels report the calendar dates of the peaks and troughs (in yy-mm format). The dates are grouped in blocks of five years. As a complement to the business and financial cycles dating tables, figures 4.1 and 4.2 sketch the evolution of the (log) industrial production and the (log) share price index for each of the countries together with the shaded construction periods to simplify visual cycle identification.

First, upon considering the descriptive statistics in the tables' upper panels, one can see that the number of real contractions fluctuates between 10 (Austria) and 14 (Portugal) over the considered sample period. We observe a slightly higher cross-country variation in the number of financial contractions ranging between 6 (Greece, Portugal) and 15 (Belgium). The average duration of real and financial contractions and expansions is asymmetric in that expansions persist longer than contractions. Also, real expansions dominate financial expansions in terms of duration. These findings confirm the preceding empirical literature (Pagan and Sossounov, 2003). Second, upon considering the calendar dates of the turning points as well as the shaded areas in the two figures, we find some casual evidence for cross-country co-movement between real and financial expansions and contractions. The dating algorithm seems able to reproduce historical recessions or stock market busts like the oil crises in the seventies, the 90-91 recession, the dotcom bubble burst and its negative real effects in the aftermath, the 9/11 terrorist attacks and their financial and real impact and last but not least the 2007 credit crisis and the resulting general economic slowdown.

Figure 4.1: Dating Business Cycles: Shaded areas represent contractionary phases.

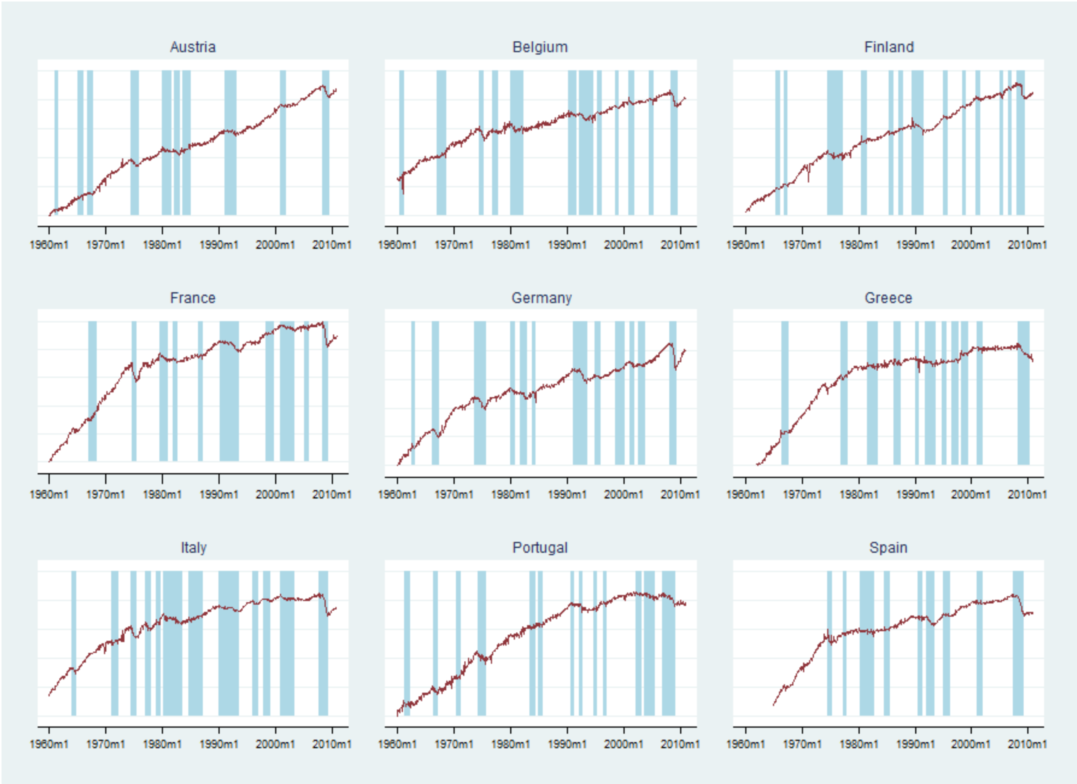

Figure 4.2: Dating Financial Cycles: Shaded areas represent bear market phases

As a benchmark for comparison with our probit business cycle synchronization framework, we estimate linear correlations on the binary dummies determined with the Bry-Boschan dating algorithm. Table 4.3 reports the results. The table distinguishes between real and financial synchronization correlations (left vs. right panels) and full sample, pre-euro and eurozone correlations (upper, middle and lower panels). As said previously, and in order to reduce the number of correlations to be reported, we only consider the correlations for France, Germany, Spain and Italy with respect to each other as well as 5 smaller countries. Thus, the table does not contain full-fledged correlation matrices but only parts of it. A number of interesting initial observations on (linear) synchronization between European countries can be made from the table. First, the magnitudes of the full sample business cycle correlations are in line with previous studies (Candelon et al., 2009). Some correlations even exceed 0.5 (e.g., France vs. Germany or Spain vs. Italy). The four large "core" countries exhibit relatively tight real links but lower links with more peripheral countries (Greece, Finland, Portugal). As for the magnitudes of the full sample financial cycle correlations, they seem marginally larger than their real counterparts (22 out of 36 financial correlations exceed their real counterparts but the differences remain limited in most cases). The question arises how stable the correlations are over time. The introduction of the single currency in 1999 constitutes a natural sample split to consider. Upon comparing subsample correlations with each other, one observes an increase in real synchronization as well as financial synchronization; but the rise in financial cycle correlations is much bigger in magnitude: eurozone financial correlations doubled or tripled as compared to their pre-eurozone counterparts. Thus, eurozone financial markets seem to have become more financially integrated after the introduction of the single currency which confirms earlier research by (Bekaert et al., 2010; Kim et al., 2005; Walti, 2011; Micco et al., 2003; Adjaoute and Danthine, 2004). This is remarkable because co-movement indicators based on tail dependence are typically unable to detect a single currency effect on systemic risk indicators (Hartmann et al., 2006). Notice that German business cycle correlations with other countries remained more or less the same after introduction of the euro which may be due to the fact that Germany was already an important driver of the surrounding countries' cycles prior to the introduction of the euro. In that sense, an important increase was probably not to be expected anyhow for pairs involving Germany given its central role as engine of European growth.

On the other hand, Italian and Spanish business cycle co-movements with the other countries did increase after the single currency was introduced.

Table 4.3: Contemporaneous Correlations (ρ)

	Business Cycles				Financial Cycles			
	France	Germany	Italy	Spain	France	Germany	Italy	Spain
Panel A: Full sample (1960M1-2010M12)								
Austria	0.412	0.456	0.268	0.410	0.474	0.372	0.472	0.285
Belgium	0.462	0.311	0.312	0.505	0.438	0.312	0.272	0.274
Finland	0.218	0.142	0.170	0.240	0.220	0.333	0.429	0.312
France		0.503	0.437	0.332		0.381	0.554	0.322
Germany	0.503		0.251	0.359	0.381		0.496	0.235
Greece	0.343	0.318	0.329	0.286	0.540	0.629	0.524	0.674
Italy	0.437	0.251		0.529	0.554	0.496		0.334
Portugal	0.089	0.167	0.099	0.187	0.507	0.730	0.776	0.648
Spain	0.332	0.359	0.529		0.322	0.235	0.334	
Panel B: Before euro intro (1960M1-1998M12)								
Austria	0.415	0.458	0.191	0.322	0.477	0.281	0.394	0.206
Belgium	0.502	0.303	0.303	0.511	0.319	0.200	0.167	0.141
Finland	0.191	0.116	0.134	0.151	0.051	0.134	0.281	0.180
France		0.479	0.396	0.344		0.277	0.497	0.194
Germany	0.479		0.192	0.359	0.277		0.378	0.068
Greece	0.340	0.300	0.334	0.214	0.357	0.515	0.419	0.497
Italy	0.396	0.192		0.480	0.497	0.378		0.249
Portugal	0.066	0.231	0.049	0.187	0.413	0.646	0.673	0.596
Spain	0.344	0.359	0.480		0.194	0.068	0.249	
Panel C: After euro intro (1999M1-2010M12)								
Austria	0.507	0.505	0.553	0.717	0.511	0.674	0.710	0.553
Belgium	0.410	0.360	0.335	0.499	0.745	0.673	0.724	0.621
Finland	0.295	0.215	0.292	0.496	0.728	0.970	0.929	0.711
France		0.546	0.621	0.310		0.699	0.799	0.671
Germany	0.546		0.456	0.360	0.699		0.900	0.740
Greece	0.376	0.373	0.318	0.495	0.676	0.763	0.651	0.833
Italy	0.621	0.456		0.682	0.799	0.900		0.640
Portugal	0.042	-0.032	0.282	0.194	0.699	0.850	0.902	0.741
Spain	0.310	0.360	0.682		0.671	0.740	0.640	

This table reports contemporaneous correlations (common sample) for different pairs of countries. The correlations for the full sample, the sample before euro introduction and the sample after euro introduction are reported.

Turning to the probit results, we estimate different nested variants of the probit model in equation (4.3). Whereas the baseline model (4.1) only reflects probit linkage between two countries' cycle dummies, one can augment the baseline case by including a single currency dummy (see equation 4.2). Finally, to address the endogeneity issue, and as a robustness check on our results, the foregoing models can also be estimated via GMM, using the lagged RHS dummy variable as instrument(s). The empirical results of all these different nested models are summarized in Tables 4.4 to 4.9. As in Table 4.3, we limit ourselves to reporting the linkages between four major EU countries (France, Spain, Germany, Italy) with respect to each other as well as 5 smaller eurozone countries. The tables report the estimated coefficients for the independent variables (cycle dummy, euro dummy) together with the corresponding marginal effects in equation (4.6)[13].

Table 4.4: Synchronization of Business and Financial Cycles - ML Estimates from probit model

	France		Germany		Italy		Spain	
	$\hat{\beta}$	MFX	$\hat{\beta}$	MFX	$\hat{\beta}$	MFX	$\hat{\beta}$	MFX
Panel A: Business Cycles								
Austria	**1.194**	**0.390**	**1.326**	**0.430**	**0.751**	**0.230**	**1.215**	**0.414**
	(0.125)	(0.042)	(0.126)	(0.042)	(0.116)	(0.036)	(0.135)	(0.047)
Belgium	**1.332**	**0.464**	**0.885**	**0.311**	**0.852**	**0.285**	**1.528**	**0.544**
	(0.124)	(0.042)	(0.119)	(0.043)	(0.113)	(0.038)	(0.139)	(0.044)
Finland	**0.630**	**0.212**	**0.414**	**0.137**	**0.467**	**0.150**	**0.713**	**0.254**
	(0.121)	(0.043)	(0.121)	(0.042)	(0.113)	(0.037)	(0.131)	(0.049)
France			**1.459**	**0.500**	**1.231**	**0.398**	**0.980**	**0.360**
			(0.125)	(0.041)	(0.118)	(0.037)	(0.132)	(0.048)
Germnay	**1.463**	**0.506**			**0.682**	**0.229**	**1.062**	**0.387**
	(0.125)	(0.041)			(0.112)	(0.038)	(0.132)	(0.048)
Greece	**0.971**	**0.345**	**0.894**	**0.317**	**0.897**	**0.303**	**0.844**	**0.311**
	(0.121)	(0.044)	(0.120)	(0.043)	(0.115)	(0.038)	(0.131)	(0.049)
Italy	**1.284**	**0.479**	**0.708**	**0.273**			**1.812**	**0.617**
	(0.123)	(0.040)	(0.116)	(0.044)			(0.159)	(0.036)
Portugal	**0.263**	**0.086**	**0.486**	**0.162**	**0.273**	**0.087**	**0.563**	**0.192**
	(0.122)	(0.041)	(0.120)	(0.042)	(0.113)	(0.037)	(0.132)	(0.048)
Spain	**0.946**	**0.306**	**1.027**	**0.333**	**1.682**	**0.452**		
	(0.127)	(0.043)	(0.128)	(0.043)	(0.147)	(0.035)		
Panel B: Financial Cycles								
Austria	**1.457**	**0.502**	**1.040**	**0.386**	**1.262**	**0.470**	**0.782**	**0.296**
	(0.128)	(0.034)	(0.114)	(0.037)	(0.109)	(0.036)	(0.113)	(0.039)
Belgium	**1.205**	**0.449**	**0.842**	**0.320**	**0.727**	**0.270**	**0.716**	**0.272**
	(0.114)	(0.039)	(0.109)	(0.040)	(0.106)	(0.038)	(0.110)	(0.041)
Finland	**0.589**	**0.223**	**0.888**	**0.332**	**1.182**	**0.410**	**0.831**	**0.314**
	(0.110)	(0.042)	(0.110)	(0.040)	(0.113)	(0.035)	(0.111)	(0.041)
France			**1.028**	**0.373**	**1.698**	**0.522**	**0.862**	**0.318**
			(0.112)	(0.039)	(0.130)	(0.032)	(0.112)	(0.040)
Germany	**1.037**	**0.388**			**1.408**	**0.476**	**0.629**	**0.238**
	(0.113)	(0.040)			(0.117)	(0.034)	(0.110)	(0.041)
Greece	**1.649**	**0.585**	**1.864**	**0.648**	**1.439**	**0.526**	**2.163**	**0.711**
	(0.182)	(0.049)	(0.175)	(0.045)	(0.159)	(0.049)	(0.198)	(0.040)
Italy	**1.748**	**0.588**	**1.434**	**0.515**			**0.921**	**0.351**
	(0.134)	(0.032)	(0.120)	(0.035)			(0.113)	(0.039)
Portugal	**1.501**	**0.545**	**2.465**	**0.766**	**2.496**	**0.787**	**1.946**	**0.669**
	(0.187)	(0.054)	(0.231)	(0.038)	(0.211)	(0.038)	(0.188)	(0.046)
Spain	**0.866**	**0.324**	**0.628**	**0.235**	**0.902**	**0.320**		
	(0.112)	(0.041)	(0.109)	(0.041)	(0.111)	(0.037)		

Notes: Probit Model - $Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t})$. S_i and S_j are business or financial cycle dummies of country i and j, respectively. The $\hat{\beta}$ estimates alongwith its marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

Estimated coefficients and corresponding marginal effects that are statistically significant at the 5 percent level are put in bold in all the tables. Moreover, we report robust standard errors beneath each estimate. Several issues will be

[13]The marginal effects for for dummies represent the change in probability of being in a recession (or bear) when the RHS dummy jumps from 0 to 1.

addressed in the discussion. First, do we find statistically and economically significant probit linkages (as reflected by probit's marginal effects)? Second, are the marginal effects (magnitude, significance) robust to changing the set of explanatory variables at the RHS of the probit model? Third (and related to the previous question), what about the marginal effects' temporal stability when considering a pre-euro sample and a euro sample? Finally, do the more general linkages tell a different story about cycle linkages than the linear correlations? To answer the latter question we compare the rank orderings of the linear and probit cyclical co-movement results.

Table 4.4 reports full sample results on business cycle synchronization (Panel A) and financial synchronization (panel B) for the baseline model (4.1). The table reports the estimated coefficient $\widehat{\beta}$ on the explanatory cycle dummy together with the corresponding marginal effects (MFX). The economic interpretation of the marginal effects is straightforward. For example, if Spain slides into recession, the probability that Italy also ends up in recession is 61.8%. The table generally provides abundant evidence for the existence of probit cycle linkages for both the business cycle and the financial cycle pairs: both the coefficient estimates $\widehat{\beta}$ and the corresponding marginal effects are found to be statistically significant at the 5 % level for most cases. Moreover, the marginal effects are usually quite large indicating that the probit-type linkages are also economically significant. The table also confirms that the dominance of financial synchronization as compared to real synchronization that we already observed in Table 4.3 seems to persist in a probit framework.

The cycle dummies that are selected to be at the LHS or RHS of the probit specification (4.3) is to some extent an arbitrary choice. Thus, the RHS dummy variable may very well also depend on the LHS dummy variable (this is almost certainly the case for the bilateral linkages between the larger European countries that exhibit important trade linkages) and as such there may exist a potential endogeneity issue and resulting estimation bias in $\widehat{\beta}$ and the resulting marginal effects (Frankel and Rose, 1998). We remedy this endogeneity issue by resorting to GMM estimation, where we use first lag of RHS country cycle as an instrument[14]. Inclusion of first lag is intuitively and economically relevant since cycles show higher level of persistence, i.e., high (auto-)correlation at lower lags. Estimation results from GMM estimation are included in Table 4.5 Largely, the coefficient and its corresponding marginal effects are statistically

[14]The $J-$test for overidentifying restrictions favors one lag.

and economically significant for most cases, the baseline results on real and financial synchronization observed in Tables 4.4 and 4.3 are not fundamentally altered. Real and financial synchronization - as reflected by the statistical and economic significance of $\widehat{\beta}$ and its marginal effects - persists; financial synchronization still dominates real synchronization (and is often even a bit higher than in the baseline case).

Table 4.5: Synchronization of Business and Financial Cycles - GMM Estimates from probit model

	France		Germany		Italy		Spain	
	$\widehat{\beta}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\beta}$	MFX
	Panel A: Business Cycles							
Austria	**1.315**	**0.429**	**1.561**	**0.501**	**0.771**	**0.236**	**1.350**	**0.460**
	(0.126)	(0.042)	(0.128)	(0.041)	(0.116)	(0.037)	(0.136)	(0.047)
Belgium	**1.413**	**0.491**	**0.987**	**0.347**	**0.887**	**0.296**	**1.713**	**0.601**
	(0.124)	(0.041)	(0.120)	(0.043)	(0.113)	(0.038)	(0.141)	(0.042)
Finland	**0.585**	**0.196**	**0.460**	**0.153**	**0.485**	**0.156**	**0.818**	**0.293**
	(0.121)	(0.043)	(0.121)	(0.042)	(0.113)	(0.037)	(0.131)	(0.049)
France			**1.579**	**0.538**	**1.381**	**0.441**	**1.119**	**0.410**
			(0.127)	(0.040)	(0.119)	(0.036)	(0.132)	(0.047)
Germnay	**1.651**	**0.565**			**0.791**	**0.266**	**1.212**	**0.440**
	(0.127)	(0.039)			(0.112)	(0.038)	(0.133)	(0.047)
Greece	**0.988**	**0.351**	**0.996**	**0.354**	**1.062**	**0.355**	**0.933**	**0.344**
	(0.121)	(0.044)	(0.121)	(0.043)	(0.116)	(0.038)	(0.131)	(0.049)
Italy	**1.307**	**0.486**	**0.714**	**0.275**			**2.115**	**0.676**
	(0.124)	(0.040)	(0.117)	(0.044)			(0.168)	(0.030)
Portugal	**0.204**	0.066	**0.458**	**0.152**	**0.249**	**0.080**	**0.620**	**0.213**
	(0.123)	(0.041)	(0.121)	(0.042)	(0.113)	(0.037)	(0.133)	(0.048)
Spain	**0.946**	**0.306**	**1.083**	**0.351**	**1.674**	**0.450**		
	(0.127)	(0.043)	(0.128)	(0.043)	(0.144)	(0.036)		
	Panel A: Business Cycles							
Austria	**1.703**	**0.560**	**1.175**	**0.429**	**1.342**	**0.496**	**0.902**	**0.338**
	(0.133)	(0.031)	(0.115)	(0.036)	(0.110)	(0.035)	(0.113)	(0.038)
Belgium	**1.195**	**0.446**	**0.890**	**0.337**	**0.624**	**0.233**	**0.803**	**0.304**
	(0.115)	(0.039)	(0.109)	(0.040)	(0.106)	(0.038)	(0.110)	(0.041)
Finland	**0.653**	**0.247**	**0.958**	**0.357**	**1.227**	**0.424**	**0.928**	**0.349**
	(0.110)	(0.042)	(0.110)	(0.039)	(0.113)	(0.035)	(0.111)	(0.040)
France			**1.110**	**0.401**	**1.670**	**0.515**	**0.918**	**0.338**
			(0.112)	(0.039)	(0.128)	(0.032)	(0.112)	(0.040)
Germnay	**1.032**	**0.386**			**1.378**	**0.467**	**0.644**	**0.244**
	(0.113)	(0.040)			(0.116)	(0.034)	(0.110)	(0.041)
Greece	**1.837**	**0.632**	**1.937**	**0.666**	**1.563**	**0.564**	**2.363**	**0.747**
	(0.188)	(0.045)	(0.177)	(0.043)	(0.161)	(0.048)	(0.204)	(0.036)
Italy	**2.168**	**0.668**	**1.632**	**0.569**			**1.009**	**0.381**
	(0.147)	(0.027)	(0.123)	(0.032)			(0.114)	(0.038)
Portugal	**1.762**	**0.615**	**2.970**	**0.828**	**2.694**	**0.820**	**2.530**	**0.791**
	(0.193)	(0.049)	(0.258)	(0.030)	(0.218)	(0.034)	(0.205)	(0.035)
Spain	**0.914**	**0.341**	**0.603**	**0.226**	**0.938**	**0.332**		
	(0.113)	(0.041)	(0.109)	(0.041)	(0.111)	(0.037)		

Notes: Probit Model - $Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t})$. S_i and S_j are business or financial cycle dummies of country i and j, respectively. The $\widehat{\beta}$ estimates alongwith its marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

Next, we investigate whether "general" probit synchronization (that is able to pick up non-linear co-movements if they exist) has been impacted by the introduction of the single currency in 1999. We do this by including an interaction term $[S_j \times EUD]$ in equations (4.1) and (4.2) where EUD is zero before

January 1999 and unity afterwards. The linear subsample results in Table 4.3 suggest the existence of such an effect but does it persist in a probit framework that also allows for non-linearity? Results are summarized in Tables 4.6 and 4.7 for estimations based on method of maximum likelihood and Tables 4.8 and 4.9 for GMM based estimations. The tables distinguish between business cycle synchronization (Tables 4.6 and 4.8) and financial cycle synchronization (Tables 4.7 and 4.9). As in all previous tables, we again consider bilateral linkages of the four major countries France, Germany, Spain and Italy with respect to 9 eurozone countries (including these four). We find that the euro dummy does not fundamentally alter the empirical fact of general (linear or non-linear) probit synchronization as it is detected in both the pre-euro sample and the euro sample. Moreover, the tables do not provide strong evidence for the existence of a generalized euro effect on bilateral business cycle synchronization because the vast majority of $\widehat{\gamma}$-estimates in the business cycle synchronization regressions is insignificantly different from zero. A few country pairs nevertheless provide some evidence for "de-coupling" business cycles ($\gamma < 0$), see, e.g., the business cycle pairs involving France w.r.t. Austria, Belgium, Greece, Italy and Spain; or Germany w.r.t. Greece and Spain in Table 4.6. Stated otherwise, the divergence results somewhat support the idea that monetary integration did not bring further economic convergence for certain countries with the core of the eurozone. Spain, Greece and Italy are indeed part of the so-called PIIGS countries whose participation in the eurozone is currently questioned by some economists[15]. Turning to the existence of a euro effect on financial cycle synchronization, $\widehat{\gamma}$ is found to be strongly significantly positive in the financial synchronization regressions. In other words, the introduction of the single currency coincides with a strong increase in financial synchronization between stock markets. This is in line with earlier literature (Artis, 2003; Beine and Candelon, 2011; Caporale and Spagnolo, 2011; Walti, 2011).

As a final empirical exercise, we assess whether the relative magnitudes of linear and probit co-movements are comparable for the same country pairs. To that aim, we rank the country pairs based on simple correlations as well as on the marginal effects obtained from various probit specifications. Next, we calculate Spearman's rank correlations, r_s, between these different rank-

[15] A theoretical story on economic divergence in monetary unions is also offered by Krugman (1991) who argues that economies of scope and scale following a monetary union can result in an increased concentration of industries whereby sector-specific shocks could become region-specific shocks and may accentuate the likelihood of asymmetric shocks leading to divergent business cycles (Kalemli-Ozcan et al., 2001).

Table 4.6: SYNCHRONIZATION OF BUSINESS CYCLES - ML ESTIMATES FROM PROBIT MODEL WITH EU DUMMY

	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX
	FRANCE				GERMANY			
Austria	**1.357**	**0.446**	**-0.456**	**-0.107**	**1.445**	**0.470**	**-0.398**	**-0.093**
	(0.145)	(0.049)	(0.040)	(0.040)	(0.141)	(0.046)	(0.042)	(0.042)
Belgium	**1.579**	**0.547**	**-0.660**	**-0.166**	**0.936**	**0.329**	**-0.171**	-0.053
	(0.148)	(0.047)	(0.041)	(0.041)	(0.135)	(0.049)	(0.062)	(0.062)
Finland	**0.612**	**0.205**	0.051	0.016	**0.365**	**0.120**	**0.161**	0.053
	(0.142)	(0.050)	(0.066)	(0.066)	(0.138)	(0.047)	(0.074)	(0.074)
France					**1.300**	**0.447**	**0.583**	**0.205**
					(0.139)	(0.047)	(0.089)	(0.089)
Germnay	**1.443**	**0.500**	0.055	0.017				
	(0.146)	(0.048)	(0.067)	(0.067)				
Greece	**1.038**	**0.369**	**-0.188**	-0.059	**0.892**	**0.317**	0.004	0.001
	(0.142)	(0.051)	(0.061)	(0.061)	(0.136)	(0.049)	(0.070)	(0.070)
Italy	**1.351**	**0.501**	**-0.180**	-0.066	**0.647**	**0.250**	**0.209**	0.081
	(0.148)	(0.047)	(0.077)	(0.077)	(0.132)	(0.051)	(0.085)	(0.085)
Portugal	**0.089**	**0.028**	**0.457**	**0.159**	**0.482**	**0.160**	0.015	0.005
	(0.149)	(0.048)	(0.080)	(0.080)	(0.137)	(0.048)	(0.069)	(0.069)
Spain	**1.013**	**0.329**	**-0.190**	-0.051	**1.046**	**0.339**	-0.060	**-0.017**
	(0.146)	(0.050)	(0.052)	(0.052)	(0.144)	(0.049)	(0.059)	(0.059)
	ITALY				SPAIN			
Austria	**0.689**	**0.210**	**0.291**	**0.092**	**1.130**	**0.384**	**0.323**	0.104
	(0.125)	(0.039)	(0.070)	(0.070)	(0.151)	(0.054)	(0.090)	(0.090)
Belgium	**0.859**	**0.287**	-0.033	-0.010	**1.626**	**0.575**	**-0.341**	-0.100
	(0.121)	(0.040)	(0.064)	(0.064)	(0.159)	(0.049)	(0.067)	(0.067)
Finland	**0.395**	**0.126**	**0.330**	0.112	**0.545**	**0.192**	**0.625**	**0.231**
	(0.122)	(0.040)	(0.075)	(0.075)	(0.149)	(0.055)	(0.101)	(0.101)
France	**1.018**	**0.330**	**1.147**	**0.423**	**0.882**	**0.323**	**0.376**	0.137
	(0.126)	(0.040)	(0.087)	(0.087)	(0.148)	(0.055)	(0.100)	(0.100)
Germnay	**0.520**	**0.174**	**0.755**	**0.279**	**1.006**	**0.367**	**0.213**	0.075
	(0.121)	(0.041)	(0.082)	(0.082)	(0.148)	(0.054)	(0.094)	(0.094)
Greece	**0.888**	**0.300**	**0.044**	0.015	**0.717**	**0.264**	**0.485**	**0.181**
	(0.122)	(0.041)	(0.068)	(0.068)	(0.147)	(0.055)	(0.102)	(0.102)
Italy					1.757	0.604	0.227	0.090
					(0.178)	(0.042)	(0.138)	(0.138)
Portugal	0.043	0.013	0.965	0.356	**0.404**	**0.135**	**0.569**	**0.203**
	(0.126)	(0.039)	(0.081)	(0.081)	(0.152)	(0.053)	(0.099)	(0.099)
Spain	**1.580**	**0.424**	**0.482**	**0.138**				
	(0.153)	(0.038)	(0.069)	(0.069)				

Notes: Probit model - ($Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t} + \gamma[S_{j,t} \times EUD])$) results. S_i and S_j are business cycle dummies of country i and j, respectively. $\hat{\beta}$ and $\hat{\gamma}$ estimates as well as marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

Table 4.7: SYNCHRONIZATION OF FINANCIAL CYCLES - ML ESTIMATES FROM PROBIT MODEL WITH EU DUMMY

	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX
	FRANCE				GERMANY			
Austia	**1.680**	**0.556**	**-0.607**	**-0.238**	**0.911**	**0.342**	**0.677**	**0.246**
	(0.160)	(0.037)	(0.086)	(0.086)	(0.122)	(0.042)	(0.081)	(0.081)
Belgium	**0.869**	**0.334**	**1.996**	**0.612**	**0.511**	**0.194**	**7.551**	**0.672**
	(0.126)	(0.046)	(0.054)	(0.054)	(0.119)	(0.044)	(0.020)	(0.020)
Finland	**0.295**	**0.111**	**1.059**	**0.403**	**0.561**	**0.215**	**2.228**	**0.652**
	(0.125)	(0.048)	(0.071)	(0.071)	(0.119)	(0.045)	(0.043)	(0.043)
France					**0.791**	**0.290**	**1.176**	**0.443**
					(0.121)	(0.044)	(0.079)	(0.079)
Germnay	**0.879**	**0.331**	**0.578**	**0.225**				
	(0.126)	(0.046)	(0.081)	(0.081)				
Greece	**1.227**	**0.459**	**0.729**	**0.284**	**1.373**	**0.320**	**7.227**	**0.651**
	(0.255)	(0.083)	(0.120)	(0.120)	(0.200)	(0.045)	(0.033)	(0.033)
Italy	**1.807**	**0.601**	**-0.183**	-0.073	**1.259**	**0.458**	**1.306**	**0.412**
	(0.158)	(0.036)	(0.099)	(0.099)	(0.127)	(0.039)	(0.078)	(0.078)
Portugal	**2.319**	**0.734**	**-0.994**	**-0.345**	**8.730**	**0.987**	**-6.556**	**-0.971**
	(0.479)	(0.083)	(0.145)	(0.145)	(0.110)	(0.004)	(0.005)	(0.005)
Spain	**0.620**	**0.233**	**0.921**	**0.355**	**0.311**	**0.117**	**1.520**	**0.539**
	(0.126)	(0.047)	(0.077)	(0.077)	(0.121)	(0.046)	(0.061)	(0.061)
	ITALY				SPAIN			
Austia	**1.166**	**0.438**	**0.629**	**0.231**	**0.711**	**0.271**	**0.275**	0.106
	(0.114)	(0.038)	(0.077)	(0.077)	(0.125)	(0.045)	(0.080)	(0.080)
Belgium	**0.439**	**0.167**	**7.850**	**0.681**	**0.353**	**0.135**	**1.624**	**0.558**
	(0.112)	(0.042)	(0.020)	(0.020)	(0.123)	(0.047)	(0.056)	(0.056)
Finland	**0.975**	**0.348**	**1.410**	**0.513**	**0.625**	**0.237**	**0.804**	**0.312**
	(0.118)	(0.039)	(0.068)	(0.068)	(0.123)	(0.046)	(0.078)	(0.078)
France	**1.518**	**0.479**	**1.206**	**0.452**	**0.600**	**0.223**	**1.040**	**0.397**
	(0.134)	(0.035)	(0.083)	(0.083)	(0.124)	(0.046)	(0.075)	(0.075)
Germnay	**1.231**	**0.426**	**1.186**	**0.446**	**0.355**	**0.134**	**1.063**	**0.404**
	(0.122)	(0.037)	(0.077)	(0.077)	(0.123)	(0.047)	(0.071)	(0.071)
Greece	**1.053**	**0.400**	**1.017**	**0.386**	**1.589**	**0.303**	**7.038**	**0.641**
	(0.186)	(0.065)	(0.090)	(0.090)	(0.235)	(0.043)	(0.036)	(0.036)
Italy					**0.863**	**0.331**	**0.216**	**0.086**
					(0.126)	(0.044)	(0.083)	(0.083)
Portugal	**2.710**	**0.823**	**-0.352**	-0.132	**2.410**	**0.769**	**-0.694**	**-0.251**
	(0.312)	(0.048)	(0.129)	(0.129)	(0.312)	(0.058)	(0.112)	(0.112)
Spain	**0.733**	**0.264**	**0.957**	**0.368**				
	(0.116)	(0.040)	(0.073)	(0.073)				

Notes: Probit model - ($Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t} + \gamma[S_{j,t} \times EUD])$) results. S_i and S_j are business cycle dummies of country i and j, respectively. $\hat{\beta}$ and $\hat{\gamma}$ estimates as well as marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

Table 4.8: SYNCHRONIZATION OF BUSINESS CYCLES WITH EURO DUMMY - GMM ESTIMATES

	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX
	FRANCE				GERMANY			
Austria	**1.527**	**0.501**	**-0.580**	**-0.127**	**1.750**	**0.562**	**-0.607**	**-0.125**
	(0.143)	(0.047)	(0.035)	(0.035)	(0.141)	(0.043)	(0.033)	(0.033)
Belgium	**1.708**	**0.586**	**-0.758**	**-0.182**	**1.069**	**0.377**	**-0.270**	-0.080
	(0.146)	(0.045)	(0.037)	(0.037)	(0.133)	(0.047)	(0.056)	(0.056)
Finland	**0.548**	**0.183**	0.100	0.032	**0.425**	**0.140**	0.116	0.037
	(0.139)	(0.049)	(0.066)	(0.066)	(0.135)	(0.047)	(0.071)	(0.071)
France					**1.446**	**0.495**	**0.480**	**0.165**
					(0.138)	(0.046)	(0.085)	(0.085)
Germany	**1.701**	**0.580**	**-0.136**	-0.040				
	(0.145)	(0.044)	(0.058)	(0.058)				
Greece	**1.064**	**0.378**	**-0.208**	-0.064	**1.025**	**0.364**	-0.093	-0.030
	(0.139)	(0.049)	(0.058)	(0.058)	(0.134)	(0.048)	(0.065)	(0.065)
Italy	**1.385**	**0.511**	**-0.207**	-0.076	**0.653**	**0.252**	**0.204**	0.079
	(0.145)	(0.046)	(0.074)	(0.074)	(0.130)	(0.050)	(0.084)	(0.084)
Portugal	-0.004	-0.001	**0.531**	**0.187**	**0.445**	**0.148**	0.042	0.013
	(0.146)	(0.046)	(0.079)	(0.079)	(0.135)	(0.047)	(0.068)	(0.068)
Spain	**1.014**	**0.329**	**-0.191**	-0.052	**1.117**	**0.362**	**-0.109**	-0.030
	(0.144)	(0.049)	(0.051)	(0.051)	(0.141)	(0.048)	(0.056)	(0.056)
	Italy				Spain			
Austria	**0.710**	**0.217**	**0.279**	0.088	**1.298**	**0.442**	**0.197**	0.060
	(0.124)	(0.039)	(0.069)	(0.069)	(0.149)	(0.052)	(0.082)	(0.082)
Belgium	**0.899**	**0.301**	-0.057	-0.018	**1.875**	**0.647**	**-0.539**	**-0.146**
	(0.120)	(0.040)	(0.063)	(0.063)	(0.159)	(0.044)	(0.056)	(0.056)
Finland	**0.414**	**0.133**	**0.317**	0.107	**0.676**	**0.240**	**0.521**	**0.190**
	(0.121)	(0.040)	(0.074)	(0.074)	(0.146)	(0.054)	(0.098)	(0.098)
France	**1.185**	**0.381**	**1.056**	**0.386**	**1.057**	**0.387**	**0.239**	0.085
	(0.126)	(0.040)	(0.089)	(0.089)	(0.145)	(0.053)	(0.094)	(0.094)
Germany	**0.645**	**0.216**	**0.678**	**0.249**	**1.195**	**0.434**	0.066	0.022
	(0.120)	(0.041)	(0.081)	(0.081)	(0.146)	(0.052)	(0.086)	(0.086)
Greece	**1.075**	**0.360**	-0.063	-0.020	**0.829**	**0.305**	**0.397**	0.147
	(0.122)	(0.040)	(0.063)	(0.063)	(0.144)	(0.054)	(0.099)	(0.099)
Italy					**2.141**	**0.681**	-0.109	-0.042
					(0.182)	(0.033)	(0.129)	(0.129)
Portugal	0.008	0.003	**0.988**	**0.364**	**0.475**	**0.160**	**0.513**	**0.181**
	(0.125)	(0.039)	(0.079)	(0.079)	(0.148)	(0.053)	(0.096)	(0.096)
Spain	**1.568**	**0.421**	**0.486**	**0.140**				
	(0.150)	(0.038)	(0.069)	(0.069)				

Notes: Probit model - ($Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t} + \gamma[S_{j,t} \times EUD])$) results. S_i and S_j are business cycle dummies of country i and j, respectively. $\hat{\beta}$ and $\hat{\gamma}$ estimates as well as marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

Table 4.9: Synchronization of Financial Cycles with Euro Dummy - GMM Estimates

	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX	$\widehat{\beta}$	MFX	$\widehat{\gamma}$	MFX
	France				Germany			
Austria	**2.112**	**0.638**	**-0.980**	**-0.369**	**1.065**	**0.393**	**0.565**	**0.209**
	(0.170)	(0.031)	(0.076)	(0.076)	(0.123)	(0.040)	(0.086)	(0.086)
Belgium	**0.867**	**0.333**	**1.998**	**0.613**	**0.562**	0.191	**10.522**	**0.672**
	(0.125)	(0.046)	(0.053)	(0.053)	(0.118)	(0.398)	(0.020)	(0.020)
Finland	**0.376**	**0.142**	**1.001**	**0.383**	**0.652**	**0.249**	**2.168**	**0.645**
	(0.123)	(0.047)	(0.071)	(0.071)	(0.119)	(0.045)	(0.046)	(0.046)
France					**0.887**	**0.324**	**1.115**	**0.423**
					(0.120)	(0.043)	(0.081)	(0.081)
Germnay	**0.872**	**0.329**	**0.583**	**0.227**				
	(0.124)	(0.045)	(0.080)	(0.080)				
Greece	**1.583**	**0.566**	**0.418**	0.165	**1.459**	**0.276**	**8.359**	**0.648**
	(0.249)	(0.069)	(0.122)	(0.122)	(0.200)	(0.156)	(0.033)	(0.033)
Italy	**2.471**	**0.716**	**-0.752**	**-0.290**	**1.487**	**0.524**	**1.140**	**0.375**
	(0.182)	(0.027)	(0.091)	(0.091)	(0.128)	(0.036)	(0.090)	(0.090)
Portugal	**4.660**	**0.921**	**-3.211**	**-0.740**	**5.023**	**0.940**	**-2.696**	**-0.710**
	(0.448)	(0.020)	(0.053)	(0.053)	(0.472)	(0.017)	(0.069)	(0.069)
Spain	**0.675**	**0.254**	**0.882**	**0.340**	**0.278**	**0.104**	**1.543**	**0.544**
	(0.124)	(0.046)	(0.077)	(0.077)	(0.120)	(0.046)	(0.060)	(0.060)
	Italy				Spain			
Austria	**1.256**	**0.467**	**0.577**	**0.214**	**0.859**	**0.323**	**0.164**	0.064
	(0.114)	(0.037)	(0.078)	(0.078)	(0.124)	(0.043)	(0.081)	(0.081)
Belgium	**0.323**	0.120	**8.856**	**0.682**	**0.469**	**0.180**	**1.544**	**0.540**
	(0.112)	(0.251)	(0.020)	(0.020)	(0.122)	(0.047)	(0.059)	(0.059)
Finland	**1.030**	**0.365**	**1.386**	**0.506**	**0.748**	**0.283**	**0.720**	**0.280**
	(0.118)	(0.038)	(0.069)	(0.069)	(0.121)	(0.045)	(0.078)	(0.078)
France	**1.494**	**0.473**	**1.215**	**0.455**	**0.674**	**0.250**	**0.990**	**0.379**
	(0.131)	(0.035)	(0.082)	(0.082)	(0.122)	(0.045)	(0.076)	(0.076)
Germnay	**1.203**	**0.418**	**1.198**	**0.450**	**0.376**	**0.142**	**1.048**	**0.399**
	(0.120)	(0.037)	(0.076)	(0.076)	(0.121)	(0.046)	(0.070)	(0.070)
Greece	**1.223**	**0.458**	**0.900**	**0.346**	**1.850**	0.251	**8.291**	**0.626**
	(0.183)	(0.061)	(0.093)	(0.093)	(0.236)	(0.663)	(0.037)	(0.037)
Italy					**0.974**	**0.369**	**0.135**	0.054
					(0.124)	(0.042)	(0.082)	(0.082)
Portugal	**3.187**	**0.883**	**-0.762**	**-0.273**	**13.174**	**1.000**	**-10.950**	**-1.000**
	(0.317)	(0.033)	(0.113)	(0.113)	(0.388)	(0.000)	(0.000)	(0.000)
Spain	**0.776**	**0.278**	**0.936**	**0.360**				
	(0.116)	(0.039)	(0.073)	(0.073)				

Notes: Probit model - ($Pr(S_{i,t} = 1) = \Phi(\alpha + \beta S_{j,t} + \gamma[S_{j,t} \times EUD])$) results. S_i and S_j are business cycle dummies of country i and j, respectively. $\hat{\beta}$ and $\hat{\gamma}$ estimates as well as marginal effects (MFX) are reported. Robust standard errors are shown in parentheses below estimates. Bold entries show significance at 5%.

ings. Results are summarized in Table 4.10 for the full sample, the pre-euro sample and the euro sample. As usual, the table further distinguishes between business and financial cycle results. The majority of correlations is found to be close to 1 and the introduction of the euro does not seem to impact the co-movements between the rankings. This does not imply that non-linear linkages are absent from the data: Only, if non-linearities exist, probit-based rankings of synchronization do not lead to fundamentally different rankings compared to correlation-based rankings.

Table 4.10: Spearman's Rank Correlations (r_s)

	$r_s(\rho, MFX_{ML})$				$r_s(\rho, MFX_{GMM})$			
	France	Germany	Italy	Spain	France	Germany	Italy	Spain
	Panel A; Business Cycles							
I	0.995	0.997	0.991	0.994	0.994	0.994	0.986	0.993
II	0.991	0.997	0.983	0.994	0.994	0.995	0.984	0.989
II	0.988	0.992	0.985	0.969	0.969	0.993	0.978	0.944
Panel B: Financial Cycles								
I	0.998	0.999	0.995	0.999	0.996	0.996	0.981	0.995
II	0.990	0.998	0.996	0.999	0.988	0.995	0.992	0.998
III	0.990	0.983	0.839	0.993	0.985	0.983	0.992	0.950

Rank correlations (r_s) for business and financial cycles. The left panel gives the comparison of contemporaneous correlation with marginal effects based on ML estimates of simple probit model. The right panel shows the similar comparison where marginal effects are based on GMM estimates. Sample periods are: (I) = 1960M1-2010M12; (II) = 1960M1-1998M12; (III) 1999M1-2010M12. The rank correlation is: $r_s(R_i, R_j) = 1 - \frac{6\sum(R_i - R_j)^2}{n(n^2-1)}$ where R is the ranking.

4.4 Conclusion

In this chapter we propose a novel approach towards measuring business cycle synchronization and financial cycle synchronization for 9 eurozone economies. The approach is based on simple probit modeling and can pick up both linear and non-linear dependence between cycles whereas the bulk of the existing literature on business and financial cycle co-movements is linear in orientation. We assess real and financial linkages between January 1960 and December 2010. Binary cycles (0/1 variables) are identified using the nonparametric Bry-Boschan dating algorithm. Next, a probit-based framework is proposed to link different countries' real and financial cycle dummies. More specifically, the framework generates marginal probit effects that are interpretable as the increase in probability of a business cycle downturn in one country (or, alternatively, a stock market bear) given an increase in probability of a business cycle downturn (or stock market bear) in another country. We believe this type of co-movement measure for business and financial cycles has more economic content for practitioners and policymakers than the traditional correlations. Moreover, if present in the data, this approach is able to pick up non-linear cyclical behavior. The probit model is further augmented with a principal component factor to control for country fundamentals driving the co-movements and a euro dummy reflecting the introduction of the single currency. We also estimate simple linear synchronization measures as a benchmark for comparison.

Using the probit framework, we find strong cross-country synchronization in both business cycles and financial cycles. This is reflected by statistically and economically significant marginal effects within the probit framework. Moreover, financial synchronization dominates business cycle synchronization in the eurozone, especially after the introduction of the single currency: whereas the euro sample coincides with a strong increase in financial synchronization, business cycle synchronization does not change much. For some country pairs (e.g., Greece, Spain or Italy relative to France and Germany), we even find some evidence of "de-coupling" business cycles but the majority of marginal business cycle effects do not change much over time. Controlling for endogeneity does not fundamentally alter our results.

Our results suggest that monetary integration has brought more financial integration; but the impact of monetary integration on business cycle synchronization remains limited or even seems to lead to a "de-coupling" of peripheral countries' business cycles relative to the core countries in a number of cases. The former observation supports the often heard plea for more international macro-prudential regulation whereas the latter observation gives ammunition to those economists that always stressed that the euro zone architecture is unfinished business and that the conditions for an optimum currency area are not fulfilled. Finally, the high rank correlations between cycle correlations and probit marginal effects suggest that the relative ranking of country pairs in terms of cyclical synchronization is robust to the use of alternative synchronization measures. Financial linkages that involve Italy and Spain constitute an exception as they exhibit lower rank correlations between the two types of cyclical co-movement measures.

Chapter 5

Conditionally Heteroskedastic Binary Choice Models for Macro-financial Time Series

5.1 Introduction

Binary choice models (BCM) are widely applied in economics and finance literature for forecasting recessions (e.g. Estrella and Hardouvelis, 1991b; Estrella and Mishkin, 1997, 1998; Dueker, 1997; Kauppi and Saikkonen, 2008), interest rate changes (e.g. Eichengreen et al., 1985; Davutyan and Parke, 1995; Frankle and Rose, 1996), assessing early warning systems for banking and currency crises (e.g. Eichengreen, Rose and Wyplosz, 1995; Goldfajn and Valdés, 1998; Demirgüç-Kunt and Detragiache, 2000; Bussière and Fratzscher, 2006; Candelon, Dumitrescu and Hurlin, 2012), predicting direction or bear conditions on the stock market (e.g., Chen, 2009; Nyberg, 2011; Candelon, Ahmed and Straetmans, 2012) etc. The crises and stock market fluctuations are however characterized by the serial dependence. Moreover, while forecasting economic crises and bears on stock market, the explanatory variables (e.g. inflation, or often used term spread) may also impact the relationship due to their random fluctuations. This is likely to plague the model with heteroskedasticity. In presence of heteroskedasticity, the parameter estimates become biased and inconsistent and may accompany large standard errors, as argued in Davidson and MacKinnon (1984). Usual consequences, like inefficiency, i.e., wide confidence intervals and wrong signals about the economic or stock market states follow and may lead to inappropriate policy or investment decisions.

To address the serial dependence, a few solutions have been proposed in the literature. Dueker (1997), for example, proposes a probit model wherein the parameters are time-varying via a first order, two state Markov process. This specification, though captures the state contingent nature of the crises data, but introduces further heteroskedasticity in the error terms as down state in the model is characterized by higher variance than the up state. Kauppi and Saikkonen (2008) on the other hand proposes an auto-regressive and dynamic extension of probit, where dependence is captured by augmenting the simple model with lags of dependent variable and the autoregressive terms of the index. Formally, this relaxes the assumption of i.i.d.-ness of disturbances of the probit model. However, while these specifications do capture the dependence structure in the probit model, they still assume homoskedastic variance frozen at unity. Besides, the model shocks might still have some residual lagged impact which may not be captured by the lagged dependent variable. For example, due to persistence (slow mean reversion), unlike lagged dependent variable, the lagged shocks carry cumulative information which is not incorporated in the former.

Another way to relax the assumption of i.i.d-ness is by letting the errors be heteroskedastic. Previously proposed solution for binary choice models is derived from the specification testing framework as proposed in, e.g., Engle (1983) and Davidson and MacKinnon (1984). In that setting, some exogenous variable(s), Z_t, which is(are) presumed to induce heteroskedasticity, replace the unit variance in the form of Harvey (1976)'s multiplicative heteroskedasticity (i.e., $exp(\lambda Z_t)$. This specification has been generally applied to the longitudinal data, where usually a bunch of explanatory variables enters the model. However, in the time series analysis, especially for forecasting recessions, crises and stock market fluctuations, the parsimony of the applied models limits the use of such an adjustment. Moreover, as pointed out in Engle (1982), specifying the exogenous causes for the changing variance ignores the fundamental fact that both mean and the variance of the model evolve together. Additionally, the identified cause, i.e., the exogenous variable(s) may not constitute an exhaustive list.

Yet another solution is proposed by Dueker (1999), where errors of ordered probit model are made conditionally heteroskedastic with state-contingent variance. The state dependent conditional variance, though well suited to the binary nature of the dependent variable, however, ignores the intermediate variations possibly brought about by the fluctuations in the independent variable.

It thus puts an artificial structure on the dynamics of the model. Furthermore, since the proposed estimation is not frequentist but Bayesian based on the Gibbs sampler, it is computationally demanding. Finally, Calzolari and Fiorentini (1998) propose a GARCH specification for the errors of Tobit model. However, unlike the BCMs, variance is estimated as a parameter in the Tobit model.

In this chapter, therefore, we propose a GRACH type conditionally heteroskedastic variance structure on the errors of the probit model. Specifically, we condition the evolution of the variance on the past information, thus bringing conditional heteroskedasticity in the short run while leaving the unconditional (long run) variance fixed (to unity). We argue that this specification will take care of the most important time series stylized facts i.e., the persistence and clustering, while at the same time allowing both mean and variance to evolve together by taking into account the past history. In terms of the estimation, our method is akin to the Bayesian data augmentation algorithm of Albert and Chib (1993). We, however, resort to the frequentist approach, i.e., maximum likelihood estimation, taking cue from Harvey, Ruiz and Sentana (1992) and Gourieroux, Monfort and Trognon (1984). The ML estimation is easy to implement and avoids the computational burden of the Bayesian approach. Data augmentation is necessary since the underlying process is only partially observable in BCM via its occurrence or non-occurrence, as determined by a threshold. Nonetheless, in most of the time series applications, these events are either exogenously determined, e.g., recession/expansion periods by NBER or CEPR; or can be extracted from the macro-financial series by some pattern recognition algorithms, e.g., Bry and Boschan (1971) or some market pressure index. The case for data augmentation becomes even stronger since the chosen explanatory variables of the model are presumed to generally move in tandem with the underlying process, e.g. the recessions. As pointed out in Davidson and MacKinnon (1984), and we demonstrate theoretically as well as via simulations, that not accounting for underlying heteroskedasticity in the probit model leads to biased estimates in both simple and dynamic BCMs. We compare the performance of heteroskedasticity augmented models with the corresponding benchmark static probit as well as dynamic specification containing lagged dependent variable. Simulation results show that extended models perform relatively better than unadjusted models. We also propose two Lagrange multiplier tests (LM_1 and LM_2) for testing the ARCH effects in BCMs. The size and power assessment of tests via simulations reveals that LM_2 test exhibits smaller size distortions and higher power properties. Moreover, we also provide empirical application(s) and show that probit models, both static as well as dynamic, when adjusted for GARCH-type conditional heteroskedasticity, provide a better fit to the time series data compared with their unadjusted counterparts.

The rest of the chapter is structured as follows. Section 5.2 presents the theoretical case for the heteroskedastic adjustment in binary choice models, develops the model, its estimation procedure as well as proposes specification tests. Section 5.3 discusses the simulation design and the results of simulations. Section 5.4 provides empirical applications of the model and section 5.5 concludes.

5.2 Heteroskedasticity in BCMs

5.2.1 Binary Choice Model

There are situations when a variable of interest, e.g., recessions, stock market conditions, monetary policy announcements etc. can best be represented by signals as to whether they are observed or not; or else whether a certain threshold is met. The signals are then translated into binary variables, where '1' depicts the observability and '0' otherwise. The underlying latent variable, y_t^*, however, remains unobserved. In the spirit of regression, the unobserved variable is related to a set of exogenous variables as,

$$y_t^* = \boldsymbol{X_t\beta} + u_t \qquad\qquad u_t \sim IID(0, \sigma^2), \tag{5.1}$$

where $\mathbf{X_t}$ is a row vector of observations on exogenous variables, their lags and the lags of dependent variables and $\boldsymbol{\beta}$ is the corresponding column vector of parameters.

Due to unobservability of the dependent variable, following censoring is employed:

$$y_t = \begin{cases} 1, & \text{if} \quad y_t^* > \tau, \\ 0, & \text{if} \quad y_t^* \leq \tau. \end{cases} \tag{5.2}$$

The threshold can be any $\tau \in \mathbb{R}$, but for simplicity, $\tau = 0$ is often employed in binary choice models. Furthermore, given the binary nature of the dependent variable, some proper continuous probability distribution such as normal, logistic, Gumbel, complementary log-log, is usually assumed. We shall stick to the normality assumption, leading to the probit model, so that we have

$$\begin{aligned} Pr(y_t = 1) &= Pr(y_t^* > 0) \\ &= Pr(\frac{u_t}{\sigma} > \frac{-\boldsymbol{X_t\beta}}{\sigma}) \\ &= Pr(\frac{u_t}{\sigma} < \frac{\boldsymbol{X_t\beta}}{\sigma}) \\ &= \Phi(\frac{\boldsymbol{X_t\beta}}{\sigma}), \end{aligned} \tag{5.3}$$

where $\Phi(.)$ is the standard cumulative normal distribution. The parameters are usually estimated by maximizing the following likelihood:

$$\mathbf{L} = \prod_{y_t=1} \Phi(\frac{\boldsymbol{X_t\beta}}{\sigma}) \prod_{y_t=0} [1 - \Phi(\frac{\boldsymbol{X_t\beta}}{\sigma})]. \tag{5.4}$$

However, the mean and variance parameters are not identified separately, requiring some normalization. This is usually achieved by freezing the variance parameter, σ^2, to unity.

However, as the time series data is characterized by the serial dependence as well as clustering, the assumptions of IID-ness in (5.1) and constant variance in (5.4) are hard to maintain. For example, the episodes of economics upturns and downturns are known to have differing variance (see e.g. Hamilton, 1989); so is the case for the stock market bull and bear periods. Since forecasting and prediction of foregoing episodes have policy relevance, not accounting for the basic features of time series data will lead to policy missteps.

Formally, as the binary response models focus on two states, the variance in two states is likely to be different. Let the variance in two states, $(y_t = 1, y_t = 0)$, be σ_1^2 and σ_0^2, respectively. By the same argument, the parameters in two states should also be different. Let these be $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_0}$. Then the estimates for $\widehat{\boldsymbol{\beta_1}}$ and $\widehat{\boldsymbol{\beta_0}}$ would be,

$$\widehat{\boldsymbol{\beta_1}} = \frac{\boldsymbol{\beta_1}}{\sigma_1} \qquad \text{and} \qquad \widehat{\boldsymbol{\beta_0}} = \frac{\boldsymbol{\beta_0}}{\sigma_0}.$$

However, in empirical applications a pooled estimate, $\widehat{\boldsymbol{\beta}}$, is assumed to govern the process. The ratio of parameters with varying variances becomes,

$$\frac{\widehat{\boldsymbol{\beta_1}}}{\widehat{\boldsymbol{\beta_0}}} = \frac{\boldsymbol{\beta}/\sigma_1}{\boldsymbol{\beta}/\sigma_0} = \frac{\sigma_0}{\sigma_1}. \tag{5.5}$$

This pooled estimate is obviously inconsistent unless variances for the states are the same. Assuming a unit variance for each state leads to standard probit, which clearly provides inconsistent estimates when the two variances are theoretically as well as empirically different. The simple probit model therefore needs to be adjusted to account for the heteroskedasticity of errors.

5.2.2 Conditionally Heteroskedastic Probit Model

One solution is to estimate separate parameters ($\boldsymbol{\beta_1}, \boldsymbol{\beta_0}$) for each state, weighted by the respective state-contingent volatility, (σ_1, σ_0). This has been suggested in Dueker (1997) and Dueker (2002), where evolution of parameters is governed by a two-state Markov-switching process. However, variance for two states remains frozen at unity in his specification. This seems implausible given the theoretical argument above as well as the empirical behavior of the time series. A rather popular solution is to assume multiplicative heteroskedasticity, $\exp(\lambda Z_t)$, where Z_t is a subset of the exogenous variables of the model, which are deemed to be causing heteroskedasticity in the error variance (see Davidson and MacKinnon, 1984; Engle, 1983). Hausman, Lo and McKinlay (1992), specify a linear specification for ordered probit, where σ_t^2 again depends on some economic variable. The problem with assigning an exogenous cause for the heteroskedasticity may pose problems as: (i) it may not be straightforward to assign the cause of changing variance (Calzolari and Fiorentini, 1998), (ii) the assigned cause may not exhaust the list of all possible causes and (iii) it ignores the fundamental fact that both mean and the variance of the process evolve together, as argued in Engle (1983).

Dueker (1999) models the conditional variance of ordered probit as evolving via the two state Markov process. Essentially, each state has been characterized by fixed variance, σ_1 and σ_0, while the conditional variance evolves as the weighted average, the weights being the probabilities of two states. This seems to be a plausible solution, however two observations can be made. For one, holding variance constant for each state still leaves the estimates inconsistent, as is obvious in (5.5). Two, the estimation is carried out in Bayesian way via Gibbs sampler, which is computationally expensive.

In this backdrop, we propose to make the error variance in binary choice models follow a generalized autoregressive conditionally heteroskedastic (GARCH) process. More specifically, we allow conditional heteroskedasticity in the short run while leaving the unconditional variance frozen to unity. Such an specification avoids assigning an external cause while at the same time allows evolution of the mean and variance based on the history of the process. Besides, it also captures the empirical feature of the time series data, such as persistence as well as clustering. Our approach is similar to Calzolari and Fiorentini (1998). They propose a GARCH specification for the errors of the Tobit model. However, the binary choice models are different from Tobit models in that the variance in the former is frozen to unity while it is estimated as a parameter in the later.

Formally, let φ_t be the information available at time t, which may contain exogenous variables and/or the lags of the dependent variable, i.e., $\varphi_t = (x_t, x_{t-1}, \ldots, x_0, y_{t-q}, y_{t-q+1}, \ldots, y_0)$. The assumptions about the short and long run variance can be written as:

$$u_t|\varphi_{t-1} \sim N(0, h_t^2) \qquad \text{but} \qquad u_t \sim N(0,1); \tag{5.6}$$

where

$$u_t|\varphi_{t-1} = h_t \times \varepsilon_t, \quad \varepsilon_t \sim N(0,1); \tag{5.7}$$

and

$$h_t^2 = \omega + \sum_{j=1}^{p} \gamma_j u_{t-j}^2 + \sum_{i=1}^{q} \delta_i h_{t-i}^2. \tag{5.8}$$

The usual constraints on the parameters of the conditional variance process, i.e., $\omega > 0$, $\gamma, \delta \geq 0$ and $\sum_{j=1}^{p} \gamma_j + \sum_{i=1}^{q} \delta_i < 1$ are assumed to ensure weak stationarity of the process and positivity of the variance (see Bollerslev, 1986).

Since GARCH(1,1) specification has been extensively applied in financial econometrics and has been found to adequately capture the underlying stylized features of the time series, we therefore propose the same for h_t^2 in (5.8) (see e.g., Hansen and Lunde, 2005). Moreover, unlike the higher order GARCH(p, q), GARC(1,1) is invariant to the temporal aggregation (Dorst and Nijman, 1993). This feature makes it ideal for applying the specification and comparing the results irrespective of the frequency of the available data. Furthermore, restricting the unconditional (long run) variance to unity but letting conditional variance to fluctuate in the short run is tantamount to volatility targeting (Engle and Mezrich, 1996). This assumption achieves the twin objectives of bringing dynamics into the variance structure and at the same time ensure identification of mean parameters in (5.3). Formally, the long run variance for GARCH(1,1) is

$$\sigma = \omega/(1 - \gamma - \delta).$$

However, for simple probit $\sigma = 1$ is assumed. Therefore letting

$$\omega = (1 - \gamma - \delta) \tag{5.9}$$

satisfies the identification condition. As a by-product of this restriction, we are left with estimating one parameter less for our model. Recently, Francq et al. (2011) report the merits of estimation of GARCH(1,1) under variance targeting and found it to perform satisfactorily in the finite samples. With this restriction, the specification for conditional variance now becomes:

$$\begin{aligned} h_t^2 &= (1 - \gamma - \delta) + \gamma u_{t-1}^2 + \delta h_{t-1}^2 \\ &= 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1). \end{aligned} \tag{5.10}$$

Using (5.10) and employing $\tau = 0$ in (5.2), we have

$$y_t = \begin{cases} 1, & \text{if} \quad \varepsilon_t > -\boldsymbol{X_t}\boldsymbol{\beta}/h_t \quad \text{and} \\ 0, & \text{otherwise.} \end{cases} \tag{5.11}$$

This can be seen as weighting the mean of latent variable by conditional volatility. Furthermore, when $\gamma,\ \delta = 0$, the model collapses to the simple probit model.

Estimation

The estimation can be carried out via the quasi-maximum likelihood. Let $\boldsymbol{\Gamma} = (\gamma, \delta)$, and $\boldsymbol{Z_t} = [(u_{t-1}^2 - 1)\ (h_{t-1}^2 - 1)]$. Let also $\pi_t = [\sqrt{1 + \boldsymbol{Z_t}\boldsymbol{\Gamma}}]^{-1}(\boldsymbol{X_t}\boldsymbol{\beta}) = (\boldsymbol{X_t}\boldsymbol{\beta})/h_t$ for time period t. Assuming normality on errors, and given that y_t being a binary variable follows the Bernoulli distribution, the conditional distribution of y_t is

$$f(y_t|\varphi_{t-1}; \boldsymbol{\Theta}) = \Phi(\pi_t)^{y_t}[1 - \Phi(\pi_t)]^{1-y_t}, \tag{5.12}$$

where $\boldsymbol{\Theta} = (\boldsymbol{\beta}', \boldsymbol{\Gamma}')$. The log-likelihood ($\mathcal{L}$) over the sample of T observations is given by

$$\mathcal{L} = \sum_{t=1}^{T} \left(y_t \log \Phi(\pi_t) + (1 - y_t) \log[1 - \Phi(\pi_t)] \right), \tag{5.13}$$

where $\Phi(.)$ is the standard cumulative normal distribution. Together with the stationary constraints and restriction (5.9), it follows from standard ARCH theory that the errors are serially uncorrelated with zeros mean and unit variance[1].

However, maximizing (5.13) poses problems because due to the latent nature of the underlying dependent variable, the errors, $\widehat{u}_t^* = y_t^* - \Phi(\widehat{\pi}_t)$, are unobservable. The estimated errors, $\widehat{u}_t = y_t - \Phi(\widehat{\pi}_t)$, on the other hand are not the true errors of the process. Furthermore, the distribution of $\widehat{u}_t$ is contaminated by being mixture of discrete and continuous variables (Albert and Chib, 1993). In view of this, two solutions are conceivable. One way is to resort to the Bayesian estimation via Albert and Chib (1993)'s data augmentation algorithm. Second is to use the conditional expectations of the errors following Harvey, Ruiz and Sentana (1992). Incidentally, following Gourieroux et al. (1984), the conditional expectations of the residuals can had from the underlying latent model (5.1), conditioned on the information set, φ_{t-1}. We follow the latter approach. This approach is appealing for at least three reasons: (i) it is analogous to the data augmentation in that it extracts conditionally consistent information about the underlying latent residuals based on the knowledge about the exogenous variables, x_t, and the binary (dummy) variable, y_t. It is emphasized that in time series applications, rather than using the criteria (5.11), the observable binary series is either endogenously determined, e.g. recession/expansion episodes by NBER or CEPR, or are extracted from relevant macro-financial series via pattern recognition algorithms, e.g., bull/bears periods via Bry and Boschan (1971). Furthermore, the explanatory variables are taken as the ones which move closely together with the underlying unobservable phenomenon, e.g., recessions/expansions. This makes data augmentation a natural choice to go about. (ii) the disturbances so estimated flow directly from the underlying unobserved process and are thus continuous. (iii) it is computationally less demanding than Bayesian estimation procedures, i.e., the Gibbs sampler approach.

From assumptions in (5.7) and (5.11) on model (5.1), it is clear that

$$y_t^* \sim TN_\tau(\boldsymbol{X_t}\boldsymbol{\beta}, h_t^2), \tag{5.14}$$

where TN_τ is the truncated normal distribution, with truncation point τ - zero in our case. Following Gourieroux et al. (1984), the disturbance term can be predicted by:

[1]See Harvey et al. (1992) for statistical properties of ARCH/GARCH process in unobserved component time series models.

$$\widetilde{u}_t = E[y_t^*|y_t, \boldsymbol{\beta}] - \boldsymbol{X_t\beta}, \tag{5.15}$$

which can be estimated as:

$$\widehat{u}_t = E[y_t^*|y_t, \widehat{\boldsymbol{\beta}}] - \boldsymbol{X_t}\widehat{\boldsymbol{\beta}}. \tag{5.16}$$

The expectation can be calculated using the properties of the truncated normal distribution - see Greene (2008)[2]. Specifically,

$$E[y_t^*|y_t = 1] = E[y_t^*|y_t^* > 0] = \boldsymbol{X_t\beta} + h_t\frac{\phi(\pi_t)}{\Phi(\pi_t)}$$
$$E[y_t^*|y_t = 0] = E[y_t^*|y_t^* < 0] = \boldsymbol{X_t\beta} - h_t\frac{\phi(\pi_t)}{1-\Phi(\pi_t)},$$

where $\Phi(.)$ and $\phi(.)$ are respectively the cumulative and the probability density functions of standard normal distribution. Therefore,

$$\begin{aligned} E[y_t^*|y_t] &= y_t\left[\boldsymbol{X_t\beta} + h_t\frac{\phi(\pi_t)}{\Phi(\pi_t)}\right] + (1-y_t)\left[\boldsymbol{X_t\beta} + \frac{\phi(\pi_t)}{1-\Phi(\pi_t)}\right] \\ &= \boldsymbol{X_t\beta} + h_t\frac{\phi(\pi_t)}{\Phi(\pi_t)[1-\Phi(\pi_t)]}[y_t - \Phi(\pi_t)]. \end{aligned} \tag{5.17}$$

Putting (5.17) in (5.16), we have

$$\widehat{u}_t = \widehat{h}_t\frac{\phi(\widehat{\pi}_t)}{\Phi(\widehat{\pi}_t)[1-\Phi(\widehat{\pi}_t)]}[y_t - \Phi(\widehat{\pi}_t)]. \tag{5.18}$$

The disturbances $\widehat{u}_t$ are called the *generalized residuals*. Gourieroux, Monfort and Trognon (1984, 1985) also show that these residuals can be used to test a variety of misspecifications of the model. Having estimated the residuals, we can proceed with th estimation via Maximum likelihood. Since we are assuming GARCH(1,1) specification, we only need one past realization for both the squared shocks as well as the conditional variance. Following McCullough and Renfro (1998), the first residual can be taken as $u_{t-1} = \frac{1}{T}\sum_{s=1}^{T}[y_s - \Phi(\boldsymbol{X_s\beta_0})]$, where $\boldsymbol{\beta_0}$ is the OLS estimate. The first value for h_{t-1}^2 may be taken as the

[2]For any $\times \sim N(\mu, \sigma^2)$ and is truncated at some constant τ,

$$E[\times|\text{truncation}] = \mu + \sigma\lambda(\alpha),$$

where $\alpha = (\tau - \mu)/\sigma$ and

$$\lambda(\alpha) = \phi(\alpha)/[1-\Phi(\alpha)] \qquad \text{if} \quad \times > \tau,$$
$$\lambda(\alpha) = -\phi(\alpha)/\Phi(\alpha) \qquad \text{if} \quad \times < \tau.$$

It is clear that for $\tau = 0, \quad \alpha = \pi_t$.

unconditional expectation of model variance, which is unity. With predicted values of volatility, $\widehat{h}_t$, the log-likelihood now becomes

$$\mathcal{L} = \sum_{t=1}^{T} \left(y_t \log \Phi(\frac{\boldsymbol{X_t \beta}}{\widetilde{h}_t}) + (1 - y_t) \log[1 - \Phi(\frac{\boldsymbol{X_t \beta}}{\widetilde{h}_t})] \right), \tag{5.19}$$

It is emphasized that by replacing h_t with $\widetilde{h}_t$, we introduce another sources of approximation. The estimates of mean and variance parameters can be obtained jointly by maximizing the likelihood function (5.19).

The asymptotics of the simple probit model under regularity conditions is well established. Therefore, the conventional large sample theory applies to the parameter estimates (Amemiya, 1985). In case the information set φ_t also contains lagged values of the dependent variable, y_{t-l}, de Jong and Woutersen (2011) show that under regularity conditions the asymptotic theory also applies to the model estimates. While we do not attempt to rigorously establish the asymptotic properties of our proposed specification, we assume that asymptotic results apply to our proposed specification as well. The reason behind such an assumption is that since we are weighting the mean equation of the model by the prediction of the conditional variance, which is positive, it should not change the asymptotic properties of the model. Based on this assumption,

$$\sqrt{T}(\widehat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}) \xrightarrow{d} \mathcal{N}(0, \Im(\boldsymbol{\Theta})^{-1}), \tag{5.20}$$

where $\Im(\boldsymbol{\Theta})$ is the Fisher information matrix (IM). It could either be computed from the Hessian of the log-likelihood (5.19), $\partial\mathcal{L}^2(\boldsymbol{\Theta})/\partial\boldsymbol{\Theta}\partial\boldsymbol{\Theta}'$, or, invoking the IM equality, as the outer product of gradient (**OPG**) of (5.19), i.e., $(\partial\mathcal{L}(\boldsymbol{\Theta})/\partial\boldsymbol{\Theta})(\partial\mathcal{L}(\boldsymbol{\Theta})/\partial\boldsymbol{\Theta}')$, all evaluated at $\widehat{\boldsymbol{\Theta}}_{ML}$. The consistency of the likelihood estimates implies the consistency of the information matrix achieved either way (see Greene, 2008).

In arriving at the above estimates of variance-covariance matrix, no model misspecification was assumed. However, misspecification can arise due to either the wrong assumption about the functional form, e.g., Logit rather than the probit, or when including the estimates or prediction of some (explanatory) variables, e.g., $\widetilde{h}_t$ in our case. The ML estimator thus becomes a *q*uasi-ML estimator. In this situation, (5.20) becomes

$$\sqrt{T}(\widehat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}) \xrightarrow{d} \mathcal{N}(0, [\mathcal{H}(\boldsymbol{\Theta})]^{-1}\Im(\boldsymbol{\Theta})[\mathcal{H}(\boldsymbol{\Theta})]^{-1}), \tag{5.21}$$

where, $\mathcal{H}(\boldsymbol{\Theta})$ is the Hessian while $\mathfrak{I}(\boldsymbol{\Theta})$ is the **OPG** based on the gradients of the log-likelihood (5.19)[3]. As before, consistent estimates of both can be obtained by evaluating these at $\widehat{\boldsymbol{\Theta}}_{ML}$. The variance-covariance matrix thus obtained is *robust* and is suggested by White (1982). With these assumptions about the asymptotic results, both the standard Wald as well as Lagrange Multiplier tests can thus be implemented for testing the null hypothesis, i.e.,

$$H_0 : \mathbf{R}\boldsymbol{\Theta} = r,$$

against the alternative

$$H_1 : \mathbf{R}\boldsymbol{\Theta} \neq r.$$

5.2.3 Specification Test for Heteroskedasticity

In this section, we propose a Lagrange Multiplier (LM) test for heteroskedasticity in the binary choice models. The LM test for testing general form of heteroskedasticity in linear models is proposed by Breusch and Pagan (1979) and White (1982). For the BCMs, such tests are suggested in Engle (1983) and Davidson and MacKinnon (1984). Engle (1982) proposed an LM test for ARCH effects while Lee (1991) suggested testing GARCH effects in linear models. We, however, drive an LM test for GARCH effects in the nonlinear binary choice models along the lines of Engle (1983) and Davidson and MacKinnon (1984). The LM test is attractive given that it requires estimation only under the null, i.e., under the restricted model. Formally, a *restricted* model is checked for the misspecification against a more general, *unrestricted*, one.

The general LM test is

$$\mathrm{LM} = \mathbf{s}(\widehat{\boldsymbol{\Theta}})'\mathbf{V}(\widehat{\boldsymbol{\Theta}})\mathbf{s}(\widehat{\boldsymbol{\Theta}}), \tag{5.22}$$

where $\mathbf{s}(\widehat{\boldsymbol{\Theta}})$ is the gradient (score) of the *unrestricted* model evaluated at *restricted* maximum likelihood parameter estimates and $\mathbf{V}(\widehat{\boldsymbol{\Theta}})$ is the asymptotic covariance matrix of the corresponding ML estimates (Greene, 2008). To emphasize the dependence, let $\pi_t(\boldsymbol{\Theta}) = \pi_t(\boldsymbol{\beta}, \boldsymbol{\Gamma}) = \boldsymbol{X_t}\boldsymbol{\beta}/\sqrt{1 + \boldsymbol{Z_t}\boldsymbol{\Gamma}}$. The first partials of (5.13) with respect to parameters $\boldsymbol{\beta}$ and $\boldsymbol{\Gamma}$ are,

[3]For derivation of the gradients and Hessian, see Appendix 5.6

$$\mathbf{s}(\boldsymbol{\beta}) = \frac{\partial \mathcal{L}(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\beta}} = \sum_{t=1}^{T}\left(\frac{y_t - \Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))}{\Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))(1-\Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})))}\phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))\frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\beta}}\right),$$
$$\mathbf{s}(\boldsymbol{\Gamma}) = \frac{\partial \mathcal{L}(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\Gamma}} = \sum_{t=1}^{T}\left(\frac{y_t - \Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))}{\Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))(1-\Phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})))}\phi(\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma}))\frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\Gamma}}\right). \tag{5.23}$$

The explicit terms for $\partial\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})/\partial\boldsymbol{\beta}$ and $\partial\pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})/\partial\boldsymbol{\Gamma}$ are:

$$\begin{aligned}\frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\beta}} &= \frac{1}{h_t}\boldsymbol{X_t}\\ \frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\Gamma}} &= -\frac{1}{2h_t^3}(\boldsymbol{X_t\beta})\boldsymbol{Z_t},\end{aligned} \tag{5.24}$$

where $\boldsymbol{Z_t} = [(u_{t-1}^2 - 1) \;\; (h_{t-1}^2 - 1)]$. Together with the assumed restrictions on ω $[= 1 - \gamma - \delta]$, testing the null of no heteroskedasticity boils down to testing parameter restrictions on γ, δ, i.e.,

$$H_0 : \gamma = \delta = 0.$$

Under the null hypothesis, the expressions in (5.24) reduce to

$$\begin{aligned}\frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\beta}}\,|_{\boldsymbol{\Gamma}=0} &= \boldsymbol{X_t}\\ \frac{\partial \pi_t(\boldsymbol{\beta},\boldsymbol{\Gamma})}{\partial \boldsymbol{\Gamma}}\,|_{\boldsymbol{\Gamma}=0} &= -\frac{1}{2}(\boldsymbol{X_t\beta})\boldsymbol{Z_t}.\end{aligned} \tag{5.25}$$

Further, $\boldsymbol{Z_t} = [(u_{t-1}^2 - 1) \quad (h_{t-1}^2 - 1)] = [(u_{t-1}^2 - 1) \quad 0]$ under the null, since the long run variance is assumed to be unity. This renders a singular variance-covariance matrix. However, under the locally equivalent alternatives, a GARCH(p, q) process can be seen as an ARCH(max(p, q)) (see, e.g., Bollerslev, 1986; Lee, 1991; Godfrey, 1991; Davidson and MacKinnon, 1993). Specifically, Lee (1991) has shown that the LM test of the null of white noise errors against a GARCH(p, q) process is equivalent to a LM test of white noise against an ARCH(p) process. We shall exploit this local equivalence and employ an ARCH(p) alternative. Consequently, $\mathbf{Z_t} = [(u_{t-1}^2 - 1) \; ... \; (u_{t-j}^2 - 1)], \quad \forall j = p$. Furthermore, we shall utilize generalized residuals (5.18), which constitute the consistent estimates of the residuals that flow directly from the underlying unobserved specification, i.e., model (5.1).

Under null hypothesis of homoskedasticity, the generalized residuals (5.18) reduce to:

$$\widehat{u}_t = \frac{\phi(\boldsymbol{X_t}\widehat{\boldsymbol{\beta}})}{\Phi(\boldsymbol{X_t}\widehat{\boldsymbol{\beta}})[1 - \Phi(\boldsymbol{X_t}\widehat{\boldsymbol{\beta}})]}[y_t - \Phi(\boldsymbol{X_t}\widehat{\boldsymbol{\beta}})].$$

Collecting the terms for the score, we have

$$\mathbf{s}(\Theta) = \sum_{t=1}^{T} \left(\frac{y_t - \Phi(\widetilde{\pi}_t)}{\Phi(\widetilde{\pi}_t)(1 - \Phi(\widetilde{\pi}_t))} \phi(\widetilde{\pi}_t) \right) \left[\boldsymbol{X_t} \quad - \frac{1}{2} \boldsymbol{X_t} \widehat{\beta} \boldsymbol{Z_t} \right]. \tag{5.26}$$

where, $\widetilde{\boldsymbol{\Theta}} = (\widehat{\mathbf{fi}}', \mathbf{0}')$, $\widetilde{\pi}_t = \pi_t(\widehat{\boldsymbol{\beta}}', \mathbf{0}')$ but now $\mathbf{Z_t} = [(u_{t-1}^2 - 1) \ ... \ (u_{t-j}^2 - 1)]$, $\forall j = p$[4].

Now that the score function can be consistently estimated, what remains from (5.22) is the consistent estimate of covariance of the parameters, $\mathbf{V}(\widetilde{\boldsymbol{\Theta}})$. There are several ways to achieve this (Greene, 2008). One can use either the inverse of Hessian of (5.13) evaluated at the ML estimates, or the BHHH estimator as the *"outer product of the gradient"*, **OPG**, or the expectation of Information matrix, or White (1982)'s sandwich estimator of the asymptotic covariance matrix of the QMLE. The first estimator in the present situation is mathematically quite involved. As one of the major consideration in developing a test statistic is the consistency and the convenience, we shall stick with the second and third estimates. This leads to two LM tests.

The first covariance matrix is an OPG estimate evaluated at ML parameters, which is given by

$$\mathbf{V_1}(\widetilde{\boldsymbol{\Theta}}) = \mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\mathbf{s}(\widetilde{\boldsymbol{\Theta}}).$$

Therefore, the first LM test statistic is then given by,

$$\mathrm{LM}_1 = \acute{\,}'\mathbf{s}(\widetilde{\boldsymbol{\Theta}})[\mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\mathbf{s}(\widetilde{\boldsymbol{\Theta}})]^{-\mathbf{1}}\mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\acute{\,} \quad \sim \chi_r^2, \tag{5.27}$$

where $\boldsymbol{\iota}$ is the $T-$ vector of *ones* and r is the number of restrictions. The expression of LM_1 without $\acute{\,}$ can be recognized as projection matrix. Therefore, statistic (5.27) can be recast as the *uncentered* R^2 from the artificial regression of ones on the score function, $s(\widetilde{\Theta})$, i.e.,

$$\boldsymbol{\iota} = \boldsymbol{s}(\widetilde{\boldsymbol{\Theta}})\boldsymbol{\lambda} + v_t, \tag{5.28}$$

as

$$\mathrm{LM}'_1 = \left(\frac{\acute{\,}'\mathbf{s}(\widetilde{\boldsymbol{\Theta}})[\mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\mathbf{s}(\widetilde{\boldsymbol{\Theta}})]^{-\mathbf{1}}\mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\acute{\,}}{\acute{\,}'\acute{\,}} \right) \acute{\,}'\acute{\,} = T \times R_u^2. \tag{5.29}$$

Therefore, once the values of scores are available, the test can be conveniently calculated by simple regression (see also Davidson and MacKinnon, 1984).

[4]For derivation, please see Appendix 5.6

The second LM test is based on the expectation of the Information Matrix[5], i.e.,

$$\mathbf{V_2}(\hat{\boldsymbol{\Theta}}) = E[\mathcal{I}(\boldsymbol{\Theta})|\varphi_{t-1}] = -E\left[\frac{\partial^2\mathcal{L}}{\partial\boldsymbol{\Theta}\partial\boldsymbol{\Theta}'}\right] = -\begin{bmatrix} \mathcal{I}_{\beta\beta} & \mathcal{I}_{\beta\Gamma} \\ \mathcal{I}_{\Gamma\beta} & \mathcal{I}_{\Gamma\Gamma} \end{bmatrix} \tag{5.30}$$

where under the null of no heteroskedasticity

$$\mathcal{I}_{\beta\beta} = E\left[\frac{\partial^2\mathcal{L}}{\partial\beta\partial\beta'}\right] = \sum_{t=1}^{T}\left[-\frac{[\phi(\widetilde{\pi}_t)]^2}{(1-\Phi(\widetilde{\pi}_t))\,\Phi(\widetilde{\pi}_t)}\right]\boldsymbol{X_t'X_t},$$

$$\mathcal{I}_{\Gamma\Gamma} = E\left[\frac{\partial^2\mathcal{L}}{\partial\Gamma\partial\Gamma'}\right] = \sum_{t=1}^{T}\left[-\frac{[\phi(\widetilde{\pi}_t)]^2}{(1-\Phi(\widetilde{\pi}_t))\,\Phi(\widetilde{\pi}_t)}\right]\frac{1}{4}(\boldsymbol{X_t}\widehat{\beta})^2\boldsymbol{Z_t'Z_t}, \tag{5.31}$$

and

$$\mathcal{I}_{\Gamma\beta} = E\left[\frac{\partial^2\mathcal{L}}{\partial\Gamma\partial\beta'}\right] = \sum_{t=1}^{T}\left[\frac{[\phi(\widetilde{\pi}_t)]^2}{(1-\Phi(\widetilde{\pi}_t))\,\Phi(\widetilde{\pi}_t)}\right]\frac{1}{2}(\boldsymbol{X_t}\widehat{\boldsymbol{\beta}})\boldsymbol{X_t'Z_t}.$$

Therefore, the second LM test boils down to:

$$\text{LM}_2 = \text{'}'\mathbf{s}(\widetilde{\boldsymbol{\Theta}})[\mathbf{V_2}(\widetilde{\boldsymbol{\Theta}})]^{-\mathbf{1}}\mathbf{s}(\widetilde{\boldsymbol{\Theta}})'\text{'} \quad \sim \chi^2_r, \tag{5.32}$$

The LM_2 can also be recast into a convenient artificial regression by noting the basic information matrix equality for $V_2(\hat{\Theta})$ in (5.32). Let $g_\beta(\pi_t) = \partial\pi_t/\partial\beta$ and $g_\Gamma(\pi_t) = \partial\pi_t/\partial\Gamma$, then

$$-E\left[\frac{\partial^2\mathcal{L}}{\partial\textbf{fi}\partial\boldsymbol{\Gamma}'}\right] = E\left[\frac{\partial\mathcal{L}}{\partial\beta}\frac{\partial\mathcal{L}}{\partial\Gamma}\right] = \sum_{t=1}^{T}\left[\frac{[\phi(\pi_t)]^2}{(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]g_\beta(\pi_t)g_\Gamma(\pi_t) \tag{5.33}$$

Evaluating terms like (5.33) leads to (5.30) (see Engle, 1983). Now, as in Engle (1983); Davidson and MacKinnon (1984), define $R_t(\widetilde{\Theta})$, whose typical element for Γ, e.g., is given by:

$$R_{\Gamma t}(\widetilde{\Theta}) = \left[\frac{\phi(\widetilde{\pi}_t)}{\sqrt{(1-\Phi(\widetilde{\pi}_t))\,\Phi(\widetilde{\pi}_t)}}\right]g_\Gamma(\widetilde{\pi}_t) \tag{5.34}$$

Similarly, define $r_t(\widetilde{\Theta})$ as

$$r_t(\widetilde{\Theta}) = \frac{y_t - \Phi(\widetilde{\pi}_t)}{\sqrt{(1-\Phi(\widetilde{\pi}_t))\,\Phi(\widetilde{\pi}_t)}}. \tag{5.35}$$

Now LM_2 can be re-written as

$$LM_2' = r_t(\widetilde{\Theta})'R_t(\widetilde{\Theta})[R_t(\widetilde{\Theta})'R_t(\widetilde{\Theta})]^{-1}R_t(\widetilde{\Theta})'r_t(\widetilde{\Theta}) \quad \sim \chi^2_r, \tag{5.36}$$

[5]Detailed derivation is given in Appendix 5.6

since $r_t(\widetilde{\Theta})'R_t(\widetilde{\Theta})$ is the restricted score (5.26) and $R_t(\widetilde{\Theta})R_t(\widetilde{\Theta})$ implies (5.33). The LM_2' is just the explained sum of squares for the OLS regression of $r_t(\widetilde{\Theta})$ on $R_t(\widetilde{\Theta})$, i.e.,

$$r_t(\widetilde{\Theta}) = R_t(\widetilde{\Theta})\zeta + \vartheta_t. \tag{5.37}$$

5.3 Simulations

5.3.1 Simulation Design

Parameter Bias and Inconsistency

In presence of heteroskedastic residuals, the parameter estimates are biased and/or inefficient and are likely to be accompanied by large standard errors. Usual consequences like inappropriate confidence intervals, wrong signals about the economic or stock market states follow and may lead to inaccurate policy or investment decisions.

In order to check the level of inconsistency/bias, we perform simulations for a range of ARCH, γ, and GARCH, δ, levels. Furthermore, we also take into consideration the fact that many of the macro-financial time series, used as exogenous variables in binary choice models, exhibit an autoregressive (AR) pattern. Examples include interest rates, growth rates of macroeconomic variables like real GDP, Industrial Production, unemployment etc. To capture this empirical fact, therefore, we generate the explanatory variable, x_t, as an AR(1) process. That said, the assumed data generating process (DGP) then is:

$$\begin{aligned} y_t^* = \alpha + \beta x_t + \xi y_{t-1}^* + u_t, \qquad &\text{where} \qquad u_t|\varphi_{t-1} \sim N(0, h_t^2), \\ u_t|\varphi_{t-1} = h_t \times \varepsilon_t, \quad &h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1), \\ &x_t = a + bx_{t-1} + \nu \qquad \text{with} \quad . \end{aligned} \tag{5.38}$$

The simulation process is as follows:

1. We use (5.38) with following set of parameters and generate the binary variable y_t using censoring (5.11):
   $(\alpha, \beta, \xi) = (0, 1, 0)$, $\gamma = (0.1, \ldots, 0.9)$, $\delta = (0.85, \ldots, 0.05)$ and $(a, b) = (0.1, 0.85)$. Moreover, in generating the DGP, following facts have also be considered:

a) The parameters for generation of y_t^* are chosen in order to avoid *complete or quasi-complete separation* and to ensure censoring between 20-90% via (5.11) (see e.g., Davidson and MacKinnon, 1993).

b) The parameters for conditional variance are chosen such that the unconditional variance is unity. Furthermore, many of the financial times series exhibit fat tails (see e.g., Taylor, 1986; Mikosch and Starica, 2000; Davis, 2010), and are not likely to admit fourth-order moment in GARCH models (since $3\gamma^2 + 2\delta\gamma + \delta^2 \geq 1$) (see e.g., Bollerslev, 1988; Francq et al., 2011). Therefore, the broad range of γ and δ values are being considered to captures this empirical feature of data as well.

2. ε_t and ν_t in (5.38) are drawn from standard normal distribution.

3. Estimation of parameters, $\widehat{\Theta} = (\alpha, \beta, \xi, \gamma, \delta)$, is done via maximum likelihood under the simple and the dynamic probit models besides their heteroskedastic counterparts.

4. Sample sizes of 100, 250, 500 and 1,000 are considered.

5. The process is replicated 1,000 times.

6. Finally, parameter estimates on $\widehat{\Theta}$ are compared to true Θ to see the bias level $(\widehat{\Theta} - \Theta)$. We report the mean values of $\widehat{\Theta}$, MP, the mean bias, MB, as well as the mean squared bias, MSB, i.e.,

$$MP = T^{-1}\sum_{t=1}^{T}\widehat{\Theta}, \quad MB = T^{-1}\sum_{t=1}^{T}(\widehat{\Theta} - \Theta) \quad \text{and} \quad MSB = T^{-1}\sum_{t=1}^{T}(\widehat{\Theta} - \Theta)^2.$$

The latter loss function corresponds to the mean squared error (MSE) of the estimates.

LM Test for ARCH Effects

In this section, we describe the simulation design for testing the ARCH effects in binary choice model. Specifically, we evaluate the size and power of the Lagrange multiplier (LM) test developed in section (5.2.3). The basic data generating process for the dependent and independent variables remains the same as in section (5.3.1). However, we consider a variety of specifications for h_t^2 in (5.38), including ARCH(1), ARCH(2) and GARCH(1,1) processes for assessing the power of LM tests, i.e.,

$$\text{ARCH(1):} \qquad h_t^2 = 1 + \gamma(u_{t-1}^2 - 1), \tag{5.39}$$

$$\text{ARCH(2):} \qquad h_t^2 = 1 + \gamma_1(u_{t-1}^2 - 1) + \gamma_2(u_{t-2}^2 - 1), \tag{5.40}$$

$$\text{GRACH(1,1):} \qquad h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1). \tag{5.41}$$

We consider a range of values for γ and δ parameters as set out in the table below.

Spec.	Parameter	Values
ARCH(1)	γ	$0.1, 0.2, \ldots, 0.9$
ARCH(2)	γ_1	$0.1, 0.2, \ldots, 0.9$
	γ_2	$1 - 0.05 - \gamma_1$
GARCH(1,1)	γ	$0.1, 0.2, \ldots, 0.9$
	δ	$0, 0.1, \ldots, 0.8$

All the parameters are chosen subject to stationarity and identification constraint set out in (5.9). The sample sizes of 100, 250, 500 and 1,000 are considered and each process is replicate 1,000 times. Furthermore, we focus on LM tests given in (5.27) and (5.32) which are based on the "outer product of the gradient" (OPG) [LM_1] and the expectation of Information matrix for the parameter covariance matrix [LM_2]. Both tests are implemented as tests for ARCH(p) effects. We report the rejection fractions at nominal values of 1%, 5% and 10%, respectively, for ARCH alternatives and at 5% for GARCH(1,1) alternative.

5.3.2 Simulation Results

Parameter Bias and Inconsistency

In this section, we discuss the results of our simulation exercise[6]. Tables 5.1, 5.2, 5.3 and 5.4 report the results for sample sizes of 100, 250, 500 and 1,000 respectively. The results are as per the simulation design of section 5.3.1 for various ARCH, γ, and GARCH, δ, levels. Columns 2-6 contain the mean of parameter values over 1,000 replications; columns 7-11 depict the mean bias of the estimates vis-a-vis the true value; while columns 12-16 report mean squared error (MSE) of the estimates. Comparing the simple (unadjusted) probit with corresponding heteroskedasticity adjusted model, it is clear that mean value of

[6] All the simulations were performed in Matlab® R2011a using Optimization toolbox's `fmincon` routine.

the $\widehat{\beta}$ is closer to the true value for the later. As regards the $\widehat{\alpha}$, simple probit yields more closer estimate on average in smaller samples sizes but the trend reverses as the sample size increases. With respect to the variance parameters (γ & δ) in the hetero-probit[7], these seem to be generally underestimated. Although the true error term is a GARCH(1,1) process, an autoregressive explanatory variable possibly interacts with the model residuals and possibly leads to disturbances with higher order GARCH effects.

[7]We shall use hetero-probit and GARCH-probit interchangeably.

Table 5.1: SIMULATION RESULTS FOR SAMPLE SIZE $T = 100$: MEAN VALUES OF PARAMETER ESTIMATES, BIASES AND MSEs IN PROBIT MODELS UNADJUSTED V/S ADJUSTED FOR CONDITIONAL HETEROSKEDASTICITY AT VARIOUS LEVELS OF ARCH/GARCH (γ/δ)

	Mean $\widehat{\Theta}$					Bias $\widehat{\Theta}$					MSE $\widehat{\Theta}$				
	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$
True values			$\alpha=0$	$\beta=1$	$\xi=0$			$\alpha=0$	$\beta=1$	$\xi=0$					
γ/δ								SIMPLE PROBIT							
0.1/0.85			-0.009	1.101				-0.009	0.101				0.046	0.087	
0.2/0.75			-0.004	1.188				-0.004	0.188				0.054	0.145	
0.3/0.65			-0.007	1.275				-0.007	0.275				0.059	0.220	
0.4/0.55			-0.008	1.369				-0.008	0.369				0.067	0.291	
0.5/0.45			0.000	1.425				0.000	0.425				0.048	0.346	
0.6/0.35			-0.001	1.517				-0.001	0.517				0.073	0.441	
0.7/0.25			-0.005	1.562				-0.005	0.562				0.065	0.489	
0.8/0.15			0.001	1.588				0.001	0.588				0.054	0.511	
0.9/0.05			0.001	1.629				0.001	0.629				0.050	0.580	
								HETERO- PROBIT							
0.1/0.85	0.213	0.183	-0.011	0.940		0.113	-0.667	-0.011	-0.060		0.103	0.499	0.037	0.132	
0.2/0.75	0.242	0.194	-0.012	0.979		0.042	-0.556	-0.012	-0.021		0.089	0.367	0.037	0.170	
0.3/0.65	0.270	0.211	-0.022	1.031		-0.030	-0.439	-0.022	0.031		0.084	0.253	0.043	0.201	
0.4/0.55	0.290	0.197	-0.033	1.101		-0.110	-0.353	-0.033	0.101		0.088	0.177	0.045	0.228	
0.5/0.45	0.280	0.212	-0.030	1.159		-0.220	-0.238	-0.030	0.159		0.113	0.106	0.050	0.256	
0.6/0.35	0.282	0.193	-0.049	1.253		-0.318	-0.157	-0.049	0.253		0.159	0.062	0.062	0.328	
0.7/0.25	0.283	0.183	-0.045	1.295		-0.417	-0.067	-0.045	0.295		0.227	0.039	0.056	0.319	
0.8/0.15	0.295	0.184	-0.046	1.300		-0.505	0.034	-0.046	0.300		0.306	0.032	0.060	0.303	
0.9/0.05	0.297	0.170	-0.048	1.338		-0.603	0.120	-0.048	0.338		0.403	0.035	0.057	0.324	
								DYNAMIC PROBIT							
0.1/0.85			0.036	1.136	-0.085			0.036	0.136	-0.085			0.112	0.112	0.221
0.2/0.75			0.012	1.214	-0.037			0.012	0.214	-0.037			0.112	0.173	0.231
0.3/0.65			0.002	1.296	-0.013			0.002	0.296	-0.013			0.131	0.251	0.252
0.4/0.55			-0.016	1.382	0.021			-0.016	0.382	0.021			0.118	0.318	0.221
0.5/0.45			-0.041	1.430	0.073			-0.041	0.430	0.073			0.124	0.371	0.255
0.6/0.35			-0.047	1.527	0.068			-0.047	0.527	0.068			0.133	0.475	0.259
0.7/0.25			-0.062	1.565	0.103			-0.062	0.565	0.103			0.136	0.518	0.289
0.8/0.15			-0.062	1.588	0.112			-0.062	0.588	0.112			0.142	0.541	0.320
0.9/0.05			-0.070	1.625	0.126			-0.070	0.625	0.126			0.146	0.592	0.303
								DYNAMIC HETERO-PROBIT							
0.1/0.85	0.225	0.171	0.042	0.956	-0.094	0.125	-0.679	0.042	-0.044	-0.094	0.118	0.506	0.091	0.161	0.164
0.2/0.75	0.244	0.176	0.030	1.024	-0.075	0.044	-0.574	0.030	0.024	-0.075	0.092	0.377	0.088	0.186	0.184
0.3/0.65	0.254	0.196	0.008	1.071	-0.054	-0.046	-0.454	0.008	0.071	-0.054	0.085	0.259	0.095	0.239	0.172
0.4/0.55	0.283	0.193	-0.003	1.128	-0.031	-0.117	-0.357	-0.003	0.128	-0.031	0.090	0.176	0.094	0.246	0.157
0.5/0.45	0.259	0.207	-0.025	1.193	0.005	-0.241	-0.243	-0.025	0.193	0.005	0.120	0.107	0.098	0.277	0.161
0.6/0.35	0.274	0.191	-0.047	1.273	0.002	-0.326	-0.159	-0.047	0.273	0.002	0.164	0.063	0.104	0.349	0.166
0.7/0.25	0.271	0.179	-0.060	1.309	0.038	-0.429	-0.071	-0.060	0.309	0.038	0.238	0.036	0.104	0.329	0.160
0.8/0.15	0.282	0.184	-0.053	1.317	0.031	-0.518	0.034	-0.053	0.317	0.031	0.320	0.029	0.098	0.338	0.153
0.9/0.05	0.282	0.171	-0.071	1.355	0.054	-0.618	0.121	-0.071	0.355	0.054	0.422	0.036	0.101	0.336	0.146

Notes: i) probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ for SIMPLE, $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$ for DYNAMIC models and $x_t = a + bx_{t-1} + \nu_t$ for all models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ for heteroskedasticity adjusted models whereas $h_t = 1$ for unadjusted models. ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) Mean parameter estimate, Mean bias, and mean squared error (*MSE*) are reported respectively in columns 2-6, 7-11 and 12-16. iii) Results are based on 1,000 simulations.

Table 5.2: SIMULATION RESULTS FOR SAMPLE SIZE $\boldsymbol{T = 250}$: MEAN VALUES OF PARAMETER ESTIMATES, BIASES AND MSEs IN PROBIT MODELS UNADJUSTED V/S ADJUSTED FOR CONDITIONAL HETEROSKEDASTICITY AT VARIOUS LEVELS OF ARCH/GARCH (γ/δ)

	Mean $\widehat{\Theta}$					Bias $\widehat{\Theta}$					MSE $\widehat{\Theta}$				
	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$
True values			$\alpha = 0$	$\beta = 1$	$\xi = 0$			$\alpha = 0$	$\beta = 1$	$\xi = 0$					
γ/δ								SIMPLE PROBIT							
0.1/0.85			0.007	1.046				0.007	0.046				0.031	0.031	
0.2/0.75			0.002	1.102				0.002	0.102				0.028	0.059	
0.3/0.65			0.001	1.166				0.001	0.166				0.023	0.089	
0.4/0.55			0.006	1.246				0.006	0.246				0.037	0.125	
0.5/0.45			0.015	1.316				0.015	0.316				0.032	0.174	
0.6/0.35			0.009	1.356				0.009	0.356				0.061	0.201	
0.7/0.25			0.001	1.410				0.001	0.410				0.037	0.235	
0.8/0.15			0.013	1.437				0.013	0.437				0.046	0.255	
0.9/0.05			0.021	1.460				0.021	0.460				0.081	0.280	
								HETERO- PROBIT							
0.1/0.85	0.201	0.248	-0.006	0.878		0.101	-0.602	-0.006	-0.122		0.072	0.440	0.012	0.091	
0.2/0.75	0.250	0.271	-0.009	0.872		0.050	-0.479	-0.009	-0.128		0.069	0.314	0.014	0.126	
0.3/0.65	0.267	0.287	-0.017	0.915		-0.033	-0.363	-0.017	-0.085		0.052	0.210	0.015	0.132	
0.4/0.55	0.299	0.277	-0.025	0.947		-0.101	-0.273	-0.025	-0.053		0.060	0.145	0.015	0.127	
0.5/0.45	0.302	0.256	-0.027	1.026		-0.198	-0.194	-0.027	0.026		0.078	0.091	0.020	0.121	
0.6/0.35	0.312	0.246	-0.042	1.067		-0.288	-0.104	-0.042	0.067		0.115	0.057	0.019	0.112	
0.7/0.25	0.312	0.222	-0.049	1.114		-0.388	-0.028	-0.049	0.114		0.179	0.036	0.021	0.112	
0.8/0.15	0.315	0.198	-0.051	1.149		-0.485	0.048	-0.051	0.149		0.259	0.025	0.024	0.100	
0.9/0.05	0.301	0.196	-0.049	1.180		-0.599	0.146	-0.049	0.180		0.378	0.040	0.025	0.107	
								DYNAMIC PROBIT							
0.1/0.85			0.002	1.051	0.003			0.002	0.051	0.003			0.042	0.036	0.074
0.2/0.75			-0.016	1.102	0.028			-0.016	0.102	0.028			0.045	0.064	0.088
0.3/0.65			-0.042	1.158	0.077			-0.042	0.158	0.077			0.048	0.094	0.103
0.4/0.55			-0.050	1.236	0.093			-0.050	0.236	0.093			0.052	0.128	0.103
0.5/0.45			-0.050	1.302	0.119			-0.050	0.302	0.119			0.067	0.176	0.125
0.6/0.35			-0.069	1.342	0.131			-0.069	0.342	0.131			0.067	0.201	0.155
0.7/0.25			-0.080	1.394	0.137			-0.080	0.394	0.137			0.072	0.231	0.192
0.8/0.15			-0.087	1.421	0.170			-0.087	0.421	0.170			0.076	0.247	0.179
0.9/0.05			-0.071	1.448	0.137			-0.071	0.448	0.137			0.082	0.276	0.202
								DYNAMIC HETERO-PROBIT							
0.1/0.85	0.203	0.229	0.013	0.889	-0.028	0.103	-0.621	0.013	-0.111	-0.028	0.080	0.458	0.026	0.095	0.056
0.2/0.75	0.234	0.257	0.003	0.896	-0.018	0.034	-0.493	0.003	-0.104	-0.018	0.064	0.324	0.029	0.120	0.057
0.3/0.65	0.257	0.275	-0.022	0.923	0.016	-0.043	-0.375	-0.022	-0.077	0.016	0.055	0.218	0.035	0.138	0.064
0.4/0.55	0.280	0.273	-0.027	0.970	0.008	-0.120	-0.277	-0.027	-0.030	0.008	0.066	0.152	0.035	0.137	0.076
0.5/0.45	0.289	0.249	-0.044	1.036	0.037	-0.211	-0.201	-0.044	0.036	0.037	0.088	0.097	0.048	0.133	0.083
0.6/0.35	0.297	0.236	-0.066	1.080	0.041	-0.303	-0.114	-0.066	0.080	0.041	0.129	0.063	0.047	0.128	0.086
0.7/0.25	0.300	0.215	-0.087	1.125	0.061	-0.400	-0.035	-0.087	0.125	0.061	0.191	0.039	0.056	0.117	0.104
0.8/0.15	0.299	0.195	-0.096	1.158	0.081	-0.501	0.045	-0.096	0.158	0.081	0.278	0.032	0.064	0.112	0.112
0.9/0.05	0.290	0.190	-0.096	1.191	0.073	-0.610	0.140	-0.096	0.191	0.073	0.394	0.042	0.062	0.116	0.109

Notes: i) probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ for SIMPLE, $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$ for DYNAMIC models and $x_t = a + bx_{t-1} + \nu_t$ for all models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ for heteroskedasticity adjusted models whereas $h_t = 1$ for unadjusted models. ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) Mean parameter estimate, Mean bias, and mean squared error (*MSE*) are reported respectively in columns 2-6, 7-11 and 12-16. iii) Results are based on 1,000 simulations.

Table 5.3: Simulation Results for sample size $T = 500$: Mean values of Parameter Estimates, Biases and MSEsin probit models unadjusted v/s adjusted for conditional heteroskedasticity at various levels of ARCH/GARCH (γ/δ)

	Mean $\widehat{\Theta}$					Bias $\widehat{\Theta}$					MSE $\widehat{\Theta}$				
	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$
True values			$\alpha = 0$	$\beta = 1$	$\xi = 0$			$\alpha = 0$	$\beta = 1$	$\xi = 0$					
γ/δ								Simple probit							
0.1/0.85			0.014	1.027				0.014	0.027				0.037	0.017	
0.2/0.75			0.020	1.080				0.020	0.080				0.063	0.032	
0.3/0.65			0.013	1.132				0.013	0.132				0.031	0.054	
0.4/0.55			0.019	1.189				0.019	0.189				0.045	0.073	
0.5/0.45			0.029	1.256				0.029	0.256				0.083	0.103	
0.6/0.35			0.022	1.290				0.022	0.290				0.058	0.122	
0.7/0.25			0.035	1.324				0.035	0.324				0.101	0.141	
0.8/0.15			0.023	1.347				0.023	0.347				0.065	0.157	
0.9/0.05			0.039	1.364				0.039	0.364				0.106	0.168	
								Hetero- probit							
0.1/0.85	0.172	0.324	-0.002	0.855		0.072	-0.526	-0.002	-0.145		0.041	0.383	0.006	0.077	
0.2/0.75	0.220	0.371	-0.010	0.832		0.020	-0.379	-0.010	-0.168		0.041	0.253	0.006	0.114	
0.3/0.65	0.269	0.387	-0.013	0.829		-0.031	-0.263	-0.013	-0.171		0.037	0.164	0.007	0.130	
0.4/0.55	0.286	0.339	-0.023	0.895		-0.114	-0.211	-0.023	-0.105		0.041	0.120	0.009	0.101	
0.5/0.45	0.292	0.301	-0.037	0.973		-0.208	-0.149	-0.037	-0.027		0.065	0.079	0.011	0.076	
0.6/0.35	0.301	0.268	-0.044	1.020		-0.299	-0.082	-0.044	0.020		0.108	0.050	0.011	0.061	
0.7/0.25	0.299	0.240	-0.050	1.061		-0.401	-0.010	-0.050	0.061		0.173	0.031	0.012	0.051	
0.8/0.15	0.304	0.212	-0.052	1.086		-0.496	0.062	-0.052	0.086		0.257	0.021	0.013	0.046	
0.9/0.05	0.306	0.194	-0.056	1.104		-0.594	0.144	-0.056	0.104		0.362	0.032	0.013	0.040	
								Dynamic probit							
0.1/0.85			0.005	1.028	0.011			0.005	0.028	0.011			0.037	0.019	0.035
0.2/0.75			-0.024	1.075	0.045			-0.024	0.075	0.045			0.027	0.035	0.047
0.3/0.65			-0.035	1.119	0.094			-0.035	0.119	0.094			0.048	0.056	0.061
0.4/0.55			-0.039	1.174	0.100			-0.039	0.174	0.100			0.048	0.072	0.078
0.5/0.45			-0.058	1.242	0.114			-0.058	0.242	0.114			0.047	0.100	0.098
0.6/0.35			-0.058	1.273	0.136			-0.058	0.273	0.136			0.059	0.117	0.113
0.7/0.25			-0.072	1.311	0.133			-0.072	0.311	0.133			0.061	0.136	0.157
0.8/0.15			-0.064	1.332	0.149			-0.064	0.332	0.149			0.078	0.151	0.176
0.9/0.05			-0.098	1.350	0.180			-0.098	0.350	0.180			0.065	0.159	0.166
								Dynamic Hetero-probit							
0.1/0.85	0.171	0.317	0.013	0.867	-0.025	0.071	-0.533	0.013	-0.133	-0.025	0.042	0.384	0.013	0.077	0.028
0.2/0.75	0.217	0.373	-0.005	0.838	-0.007	0.017	-0.377	-0.005	-0.162	-0.007	0.040	0.251	0.015	0.116	0.030
0.3/0.65	0.259	0.379	-0.019	0.837	0.017	-0.041	-0.271	-0.019	-0.163	0.017	0.038	0.170	0.020	0.133	0.041
0.4/0.55	0.272	0.343	-0.035	0.901	0.030	-0.128	-0.207	-0.035	-0.099	0.030	0.047	0.126	0.026	0.107	0.051
0.5/0.45	0.279	0.303	-0.062	0.976	0.053	-0.221	-0.147	-0.062	-0.024	0.053	0.073	0.090	0.037	0.083	0.068
0.6/0.35	0.298	0.260	-0.092	1.015	0.083	-0.302	-0.090	-0.092	0.015	0.083	0.112	0.055	0.047	0.071	0.085
0.7/0.25	0.295	0.234	-0.118	1.061	0.111	-0.405	-0.016	-0.118	0.061	0.111	0.181	0.039	0.061	0.057	0.108
0.8/0.15	0.301	0.205	-0.130	1.085	0.115	-0.499	0.055	-0.130	0.085	0.115	0.263	0.028	0.066	0.053	0.117
0.9/0.05	0.306	0.183	-0.158	1.098	0.156	-0.594	0.133	-0.158	0.098	0.156	0.365	0.032	0.078	0.046	0.137

Notes: i) probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ for Simple, $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$ for Dynamic models and $x_t = a + bx_{t-1} + \nu_t$ for all models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ for heteroskedasticity adjusted models whereas $h_t = 1$ for unadjusted models. ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) Mean parameter estimate, Mean bias, and mean squared error (*MSE*) are reported respectively in columns 2-6, 7-11 and 12-16. iii) Results are based on 1,000 simulations.

Table 5.4: Simulation Results for sample size $T = 1,000$: Mean values of Parameter Estimates, Biases and MSEs in probit models unadjusted v/s adjusted for conditional heteroskedasticity at various levels of ARCH/GARCH (γ/δ)

	Mean $\widehat{\Theta}$					Bias $\widehat{\Theta}$					MSE $\widehat{\Theta}$				
	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$
True values			$\alpha = 0$	$\beta = 1$	$\xi = 0$			$\alpha = 0$	$\beta = 1$	$\xi = 0$					
γ/δ								Simple probit							
0.1/0.85			0.022	1.019				0.022	0.019				0.053	0.008	
0.2/0.75			0.033	1.058				0.033	0.058				0.080	0.017	
0.3/0.65			0.030	1.109				0.030	0.109				0.072	0.031	
0.4/0.55			0.045	1.167				0.045	0.167				0.113	0.047	
0.5/0.45			0.071	1.200				0.071	0.200				0.178	0.062	
0.6/0.35			0.056	1.235				0.056	0.235				0.133	0.077	
0.7/0.25			0.061	1.257				0.061	0.257				0.146	0.088	
0.8/0.15			0.070	1.269				0.070	0.269				0.170	0.096	
0.9/0.05			0.077	1.278				0.077	0.278				0.171	0.101	
								Hetero- probit							
0.1/0.85	0.150	0.407	-0.002	0.829		0.050	-0.443	-0.002	-0.171		0.023	0.325	0.003	0.076	
0.2/0.75	0.218	0.467	-0.009	0.764		0.018	-0.283	-0.009	-0.236		0.020	0.192	0.003	0.122	
0.3/0.65	0.257	0.432	-0.018	0.796		-0.043	-0.218	-0.018	-0.204		0.019	0.137	0.004	0.110	
0.4/0.55	0.293	0.325	-0.032	0.893		-0.107	-0.225	-0.032	-0.107		0.023	0.104	0.006	0.059	
0.5/0.45	0.297	0.270	-0.038	0.953		-0.203	-0.180	-0.038	-0.047		0.051	0.069	0.007	0.037	
0.6/0.35	0.305	0.230	-0.048	0.989		-0.295	-0.120	-0.048	-0.011		0.094	0.034	0.008	0.030	
0.7/0.25	0.306	0.202	-0.052	1.013		-0.394	-0.048	-0.052	0.013		0.161	0.010	0.008	0.018	
0.8/0.15	0.306	0.196	-0.048	1.024		-0.494	0.046	-0.048	0.024		0.250	0.008	0.008	0.018	
0.9/0.05	0.306	0.188	-0.046	1.028		-0.594	0.138	-0.046	0.028		0.357	0.022	0.007	0.018	
								Dynamic probit							
0.1/0.85			-0.001	1.016	0.024			-0.001	0.016	0.024			0.024	0.010	0.021
0.2/0.75			-0.020	1.048	0.070			-0.020	0.048	0.070			0.046	0.018	0.032
0.3/0.65			-0.042	1.094	0.097			-0.042	0.094	0.097			0.028	0.031	0.050
0.4/0.55			-0.050	1.151	0.125			-0.050	0.151	0.125			0.052	0.044	0.061
0.5/0.45			-0.043	1.185	0.143			-0.043	0.185	0.143			0.085	0.059	0.098
0.6/0.35			-0.067	1.223	0.140			-0.067	0.223	0.140			0.045	0.072	0.121
0.7/0.25			-0.069	1.244	0.167			-0.069	0.244	0.167			0.072	0.082	0.137
0.8/0.15			-0.085	1.260	0.182			-0.085	0.260	0.182			0.061	0.090	0.160
0.9/0.05			-0.088	1.271	0.184			-0.088	0.271	0.184			0.069	0.097	0.194
								Dynamic Hetero-probit							
0.1/0.85	0.157	0.397	0.002	0.832	-0.011	0.057	-0.453	0.002	-0.168	-0.011	0.026	0.329	0.007	0.075	0.015
0.2/0.75	0.213	0.466	-0.014	0.767	0.010	0.013	-0.284	-0.014	-0.233	0.010	0.020	0.193	0.009	0.124	0.019
0.3/0.65	0.241	0.447	-0.033	0.801	0.032	-0.059	-0.203	-0.033	-0.199	0.032	0.022	0.137	0.013	0.112	0.028
0.4/0.55	0.278	0.350	-0.063	0.886	0.059	-0.122	-0.200	-0.063	-0.114	0.059	0.030	0.108	0.024	0.069	0.046
0.5/0.45	0.290	0.285	-0.101	0.943	0.100	-0.210	-0.165	-0.101	-0.057	0.100	0.057	0.074	0.037	0.044	0.066
0.6/0.35	0.299	0.236	-0.138	0.983	0.137	-0.301	-0.114	-0.138	-0.017	0.137	0.101	0.044	0.051	0.035	0.088
0.7/0.25	0.309	0.201	-0.170	1.002	0.178	-0.391	-0.049	-0.170	0.002	0.178	0.160	0.017	0.063	0.023	0.107
0.8/0.15	0.317	0.185	-0.188	1.009	0.201	-0.483	0.035	-0.188	0.009	0.201	0.241	0.013	0.073	0.024	0.133
0.9/0.05	0.319	0.176	-0.207	1.010	0.229	-0.581	0.126	-0.207	0.010	0.229	0.345	0.023	0.082	0.023	0.151

Notes: i) probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ for Simple, $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$ for Dynamic models and $x_t = a + bx_{t-1} + v_t$ for all models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ for heteroskedasticity adjusted models whereas $h_t = 1$ for unadjusted models. ε_t & $v_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) Mean parameter estimate, Mean bias, and mean squared error (*MSE*) are reported respectively in columns 2-6, 7-11 and 12-16. iii) Results are based on 1,000 simulations.

As can be expected, the $\widehat{\alpha}$ and $\widehat{\beta}$ in both unadjusted and adjusted models tend to the true value as the sample size grows. However, the estimates under hetero-probit approach the true value at a faster rate than simple model. Correspondingly, the level of bias in the estimates becomes smaller in the hetero-probit as the sample becomes larger. Looking at the behavior of the estimates in terms of ARCH/GARCH levels, it is easily seen that the level of bias in both models increases as the level of ARCH (γ) parameter increases. However, the rate of divergence of estimates from their true values is higher for simple probit than the hetero-probit.

Since the crises and the stock market fluctuations are characterized by the serial dependence, we also estimate a dynamic version of the simple probit with corresponding heteroskedasticity adjusted counterpart. The dynamics is introduced by including the first lag of dummy (dependent) variable. Two lower panels in Tables 5.1, 5.2, 5.3 and 5.4 contain the results. The story with respect to the mean values of the estimates as well as the bias, both based on the sample size as well as the ARCH levels, is similar to that of simple versus the hetero-probit. However, it may be remarked that in the dynamic hetero-probit model, the level of bias in $\widehat{\beta}$ for sample sizes of 100 and 250 is slightly higher than the simple probit model.

Table 5.5: Simulations results: Mean Squared Error Ratios between probit models Unadjusted vs Adjusted for Conditional Heteroskedasticity in model Residuals

T	100		250		500		1000	
γ	S-H	D-DH	S-H	D-DH	S-H	D-DH	S-H	D-DH
0.1	0.66	0.70	0.34	0.38	0.21	0.24	0.11	0.13
0.2	0.86	0.93	0.46	0.53	0.29	0.30	0.14	0.15
0.3	1.09	1.05	0.68	0.69	0.42	0.42	0.28	0.27
0.4	1.27	1.29	0.99	0.94	0.72	0.67	0.80	0.64
0.5	1.35	1.34	1.44	1.32	1.35	1.20	1.70	1.34
0.6	1.35	1.36	1.80	1.57	1.99	1.66	2.59	2.04
0.7	1.53	1.57	2.11	1.98	2.76	2.40	4.79	3.62
0.8	1.69	1.60	2.55	2.20	3.41	2.86	5.24	3.81
0.9	1.79	1.76	2.61	2.37	4.19	3.43	5.67	4.15

i) S-H = Simple vs Hetero-probit; D-DH = Dynamic vs Dynamic Hetero-probit. ii) See footnotes in tables 5.1, 5.2, 5.3 and 5.4 for model description. iii) Results are based on 1,000 simulations.

It is clear that in presence of heteroskedasticity, both adjusted as well as unadjusted models fetch biased estimates, some less biased some more. As remarked above, the presence of an AR type exogenous variable possibly interacts with the error term in such a way that a higher order GARCH assumption might be needed for the model residuals. Finally, recognizing the fact that of the two competing estimators, one with smaller variance around the true value (i.e., a smaller mean squared error, MSE) is the efficient and preferable, we also computed the MSEs of the estimates. Columns 12-16 of the tables under review report the results. As with mean biases, the MSEs of heteroskedasticity augmented models are generally lower than the un-augmented models. Surprisingly, the value addition of the augmentation is smaller at lower ARCH levels even for higher sample sizes. However, at higher ARCH levels, the gain is substantial. For a clearer view therefore, Table 5.5 reports the gain/loss of heteroskedasticity augmentation as a ratio of the unadjusted to adjusted model. An obvious pattern of increasing pay-off at higher levels of ARCH as well as sample size is quite apparent. Overall, both criteria show that accounting for heteroskedasticity when it indeed is present, leads to a relatively efficient estimator. Moreover, as many of the macro-financial time series used as exogenous variables in the predictive models exhibit fat-tails, the efficiency gain at higher levels of ARCH, viz., when assumption about finite fourth moment is likely to be violated, further underscores the utility of heteroskedastic adjustment in binary choice models.

LM Test for ARCH Effects

This section elaborates the outcome of simulations for the size and power of the LM tests described in section (5.2.3). The results under the null of homoskedastic residuals are reported in Table 5.6. From the results for the size of tests, it is obvious that the one based on OPG covariance estimate (LM_1) suffers from the well known problem of being hugely oversized. On the other hand, the one based on the expected Information matrix (LM_2) is slightly oversized for small samples while slightly undersized for large samples. Davidson and MacKinnon (1984) also find similar performance of the two tests. They argue that the better performance of LM_2 compared with the LM_1 is due to the fact that the former is based on more efficient estimate of covariance matrix. Consequently, it is less likely to commit Type-I error when LM_2 is used.

The results for the power of two tests are reported in Tables 5.7, 5.8 and 5.9 for the alternatives that the errors follow, respectively, the ARCH(1), ARCH(2) and GARCH(1,1) processes. For ARCH(1) alternative, unlike the size results,

Table 5.6: Simulations: Size of Lagrange Multiplier (LM) Test for ARCH effects in Binary Choice Models- rejection fractions

	LM1			LM2		
	1%	5%	10%	1%	5%	10%
T						
	Simple probit					
100	0.071	0.201	0.291	0.013	0.029	0.040
250	0.087	0.160	0.234	0.013	0.037	0.060
500	0.055	0.138	0.197	0.015	0.046	0.085
1000	0.064	0.113	0.167	0.033	0.056	0.090
	Dynamic probit					
100	0.086	0.225	0.319	0.012	0.028	0.053
250	0.069	0.140	0.202	0.019	0.039	0.069
500	0.048	0.127	0.186	0.011	0.025	0.052
1000	0.077	0.139	0.192	0.028	0.060	0.106

Notes: i) Probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ and $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$, for Simple and Dynamic models, respectively and $x_t = a + bx_{t-1} + \nu_t$. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1)$ for heteroskedasticity adjusted models while $h_t^2 = 1$ for unadjusted ones. For all models, ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) LM_1 and LM_2 is based on (5.27) and (5.32), respectively. iii) Rejection proportions under the null of $\gamma = 0$ are reported. iv) Results are based on 1,000 simulations.

LM_1 shows considerably greater power for small samples at all nominal levels. However, for large samples, LM_2 has higher power than LM_1. While it is an empirical fact that the power of a test increases with sample size, however, the gain in power by LM_2 for large samples sizes is higher than LM_1 for both simple as well as dynamic probit models. The power of both tests also increases as we move farther from the null, i.e., for the higher ARCH levels. For ARCH(2) alternative again, LM_1 has higher empirical power than LM_2 for lower sample sizes; but the story is otherwise at large samples. On the other hand, when GARCH(1,1) alternative is considered[8], LM_1 test exhibits poor performance. While its power increases with the increase in ARCH level; surprisingly, it does not increase with the sample size. The LM_2 test on the other hand shows power increments for both higher ARCH and GARCH levels as well as the sample sizes.

[8]We report results for only hetero-probit model and 5% nominal level. The results for dynamic hetero-probit and other nominal levels are quite similar to hetero-probit. These results are not reported for space considerations.

Table 5.7: SIMULATIONS: POWER OF LAGRANGE MULTIPLIER (LM) TEST FOR ARCH EFFECTS IN BINARY CHOICE MODELS- REJECTION FRACTIONS WHEN $h_t^2 \sim ARCH(1)$

	LM_1			LM_2			LM_1			LM_2		
	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
	PANEL A: HETERO- PROBIT											
γ	T = 100						T = 250					
0.1	0.082	0.195	0.291	0.019	0.049	0.065	0.071	0.161	0.222	0.029	0.063	0.094
0.2	0.069	0.183	0.253	0.019	0.052	0.077	0.072	0.138	0.205	0.052	0.093	0.124
0.3	0.070	0.173	0.251	0.033	0.062	0.090	0.056	0.128	0.197	0.066	0.125	0.168
0.4	0.073	0.203	0.277	0.040	0.079	0.107	0.053	0.121	0.205	0.087	0.164	0.218
0.5	0.091	0.202	0.290	0.048	0.079	0.112	0.060	0.134	0.199	0.145	0.202	0.249
0.6	0.098	0.220	0.319	0.066	0.111	0.137	0.059	0.157	0.245	0.162	0.237	0.286
0.7	0.107	0.242	0.340	0.079	0.132	0.177	0.082	0.192	0.283	0.209	0.284	0.334
0.8	0.166	0.314	0.397	0.102	0.152	0.211	0.151	0.262	0.354	0.218	0.298	0.365
0.9	0.323	0.460	0.556	0.100	0.164	0.248	0.344	0.482	0.564	0.275	0.430	0.515
	T = 500						T = 1000					
0.1	0.056	0.110	0.158	0.035	0.072	0.103	0.036	0.083	0.150	0.055	0.102	0.159
0.2	0.052	0.107	0.160	0.058	0.118	0.165	0.031	0.088	0.154	0.133	0.206	0.261
0.3	0.039	0.094	0.164	0.113	0.183	0.229	0.034	0.093	0.191	0.202	0.298	0.374
0.4	0.036	0.115	0.188	0.180	0.276	0.322	0.037	0.125	0.228	0.283	0.386	0.454
0.5	0.043	0.115	0.217	0.242	0.316	0.395	0.049	0.149	0.272	0.372	0.481	0.570
0.6	0.044	0.137	0.246	0.287	0.377	0.432	0.067	0.200	0.350	0.480	0.581	0.629
0.7	0.069	0.173	0.288	0.352	0.453	0.503	0.105	0.258	0.398	0.554	0.650	0.701
0.8	0.143	0.269	0.375	0.394	0.488	0.557	0.181	0.324	0.493	0.644	0.731	0.779
0.9	0.380	0.497	0.595	0.549	0.680	0.745	0.530	0.676	0.751	0.816	0.892	0.917
	PANEL B: DYNAMIC HETERO- PROBIT											
	T = 100						T = 250					
0.1	0.087	0.204	0.285	0.018	0.037	0.066	0.074	0.156	0.234	0.015	0.042	0.076
0.2	0.085	0.207	0.300	0.021	0.051	0.073	0.066	0.138	0.212	0.038	0.084	0.114
0.3	0.085	0.213	0.298	0.026	0.059	0.089	0.062	0.140	0.223	0.066	0.118	0.167
0.4	0.083	0.210	0.304	0.045	0.092	0.125	0.040	0.136	0.211	0.082	0.143	0.191
0.5	0.097	0.239	0.333	0.032	0.066	0.103	0.064	0.160	0.240	0.120	0.179	0.233
0.6	0.117	0.254	0.345	0.051	0.099	0.140	0.066	0.162	0.243	0.123	0.217	0.278
0.7	0.136	0.294	0.396	0.064	0.118	0.150	0.104	0.201	0.306	0.187	0.262	0.311
0.8	0.203	0.358	0.457	0.065	0.119	0.188	0.166	0.281	0.393	0.218	0.314	0.371
0.9	0.362	0.506	0.614	0.069	0.159	0.251	0.360	0.492	0.590	0.301	0.443	0.525
	T = 500						T = 1000					
0.1	0.047	0.115	0.188	0.039	0.078	0.114	0.039	0.096	0.159	0.043	0.096	0.147
0.2	0.031	0.095	0.168	0.059	0.122	0.177	0.033	0.082	0.159	0.102	0.184	0.237
0.3	0.052	0.128	0.193	0.120	0.187	0.245	0.035	0.106	0.199	0.160	0.242	0.305
0.4	0.051	0.140	0.236	0.158	0.241	0.301	0.050	0.157	0.264	0.245	0.344	0.403
0.5	0.052	0.149	0.240	0.204	0.283	0.346	0.073	0.185	0.299	0.369	0.465	0.525
0.6	0.069	0.172	0.274	0.277	0.359	0.414	0.079	0.216	0.342	0.418	0.526	0.593
0.7	0.091	0.212	0.327	0.322	0.416	0.472	0.140	0.298	0.433	0.515	0.626	0.664
0.8	0.162	0.302	0.410	0.383	0.472	0.537	0.192	0.400	0.536	0.617	0.712	0.759
0.9	0.410	0.544	0.646	0.537	0.662	0.722	0.558	0.693	0.763	0.817	0.874	0.902

Notes: i) Probit Model: $Pr(y_t = 1) = \Phi(\pi_t / h_t)$, where $\pi_t = \alpha + \beta x_t$ and $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$, for SIMPLE and DYNAMIC models, respectively and $x_t = a + b x_{t-1} + \nu_t$. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1)$ for heteroskedasticity adjusted models while $h_t^2 = 1$ for unadjusted ones. For all models, ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) LM_1 and LM_2 is based on (5.27) and (5.32), respectively. iii) Rejection proportions under the null of $\gamma \neq 0$ are reported. iv) Results are based on 1,000 simulations.

Table 5.8: SIMULATIONS: POWER OF LAGRANGE MULTIPLIER (LM) TEST FOR ARCH EFFECTS IN BINARY CHOICE MODELS- REJECTION FRACTIONS WHEN $h_t^2 \sim ARCH(2)$

		LM_1			LM_2			LM_1			LM_2		
		1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
ARCH		Panel A: Hetero-Probit											
γ_1	γ_2	T = 100						T = 250					
0.1	0.85	0.537	0.693	0.776	0.060	0.101	0.155	0.589	0.710	0.780	0.281	0.488	0.589
0.2	0.75	0.453	0.621	0.716	0.070	0.113	0.158	0.583	0.699	0.760	0.293	0.482	0.597
0.3	0.65	0.477	0.633	0.717	0.073	0.109	0.152	0.571	0.693	0.752	0.294	0.486	0.608
0.4	0.55	0.505	0.651	0.743	0.073	0.123	0.173	0.591	0.693	0.761	0.294	0.498	0.611
0.5	0.45	0.491	0.629	0.717	0.081	0.124	0.193	0.566	0.682	0.747	0.294	0.484	0.613
0.6	0.35	0.496	0.639	0.724	0.086	0.137	0.186	0.579	0.699	0.768	0.322	0.515	0.618
0.7	0.25	0.500	0.658	0.744	0.097	0.151	0.193	0.599	0.704	0.765	0.330	0.540	0.643
0.8	0.15	0.538	0.697	0.768	0.079	0.131	0.183	0.639	0.752	0.804	0.334	0.542	0.657
0.9	0.05	0.558	0.716	0.787	0.098	0.144	0.202	0.677	0.767	0.806	0.338	0.575	0.694
		T = 500						T = 1000					
0.1	0.85	0.681	0.775	0.819	0.650	0.783	0.828	0.791	0.868	0.917	0.936	0.969	0.977
0.2	0.75	0.654	0.767	0.817	0.680	0.807	0.857	0.760	0.845	0.89	0.918	0.957	0.970
0.3	0.65	0.666	0.755	0.819	0.711	0.816	0.862	0.751	0.837	0.889	0.923	0.961	0.973
0.4	0.55	0.653	0.748	0.806	0.682	0.826	0.870	0.742	0.839	0.894	0.922	0.956	0.974
0.5	0.45	0.620	0.728	0.800	0.683	0.807	0.860	0.761	0.861	0.901	0.951	0.972	0.984
0.6	0.35	0.682	0.786	0.837	0.727	0.848	0.887	0.791	0.865	0.905	0.936	0.970	0.980
0.7	0.25	0.695	0.782	0.826	0.733	0.860	0.902	0.792	0.868	0.907	0.933	0.965	0.978
0.8	0.15	0.722	0.814	0.864	0.729	0.837	0.892	0.829	0.894	0.926	0.964	0.980	0.987
0.9	0.05	0.739	0.822	0.868	0.757	0.873	0.913	0.844	0.909	0.935	0.957	0.974	0.985
		Panel B: Dynamic Hetero- Probit											
		T = 100						T = 250					
0.1	0.85	0.519	0.680	0.756	0.060	0.104	0.165	0.613	0.723	0.779	0.294	0.496	0.602
0.2	0.75	0.494	0.655	0.738	0.075	0.122	0.176	0.599	0.713	0.779	0.291	0.495	0.598
0.3	0.65	0.477	0.623	0.723	0.070	0.106	0.161	0.601	0.706	0.775	0.297	0.503	0.627
0.4	0.55	0.486	0.647	0.751	0.083	0.141	0.200	0.562	0.697	0.766	0.314	0.504	0.601
0.5	0.45	0.476	0.646	0.718	0.096	0.141	0.199	0.562	0.686	0.755	0.327	0.531	0.625
0.6	0.35	0.498	0.653	0.733	0.086	0.128	0.185	0.612	0.714	0.769	0.341	0.522	0.629
0.7	0.25	0.511	0.665	0.746	0.100	0.137	0.209	0.621	0.738	0.788	0.314	0.526	0.630
0.8	0.15	0.512	0.663	0.739	0.092	0.138	0.188	0.624	0.74	0.799	0.331	0.543	0.641
0.9	0.05	0.549	0.684	0.766	0.079	0.122	0.177	0.648	0.752	0.811	0.347	0.552	0.659
		T = 500						T = 1000					
0.1	0.85	0.677	0.776	0.832	0.675	0.812	0.850	0.793	0.865	0.912	0.945	0.971	0.98
0.2	0.75	0.626	0.734	0.788	0.667	0.790	0.848	0.778	0.847	0.882	0.937	0.965	0.974
0.3	0.65	0.649	0.751	0.803	0.687	0.814	0.856	0.733	0.829	0.878	0.925	0.959	0.969
0.4	0.55	0.649	0.758	0.807	0.681	0.813	0.864	0.767	0.853	0.897	0.935	0.967	0.978
0.5	0.45	0.661	0.759	0.803	0.703	0.841	0.878	0.779	0.865	0.899	0.949	0.966	0.975
0.6	0.35	0.670	0.768	0.826	0.709	0.829	0.882	0.809	0.888	0.922	0.957	0.979	0.984
0.7	0.25	0.674	0.754	0.809	0.722	0.845	0.880	0.778	0.867	0.909	0.949	0.975	0.98
0.8	0.15	0.701	0.785	0.842	0.708	0.842	0.882	0.830	0.889	0.940	0.947	0.973	0.985
0.9	0.05	0.716	0.801	0.846	0.727	0.863	0.895	0.861	0.922	0.941	0.961	0.990	0.992

Notes: i) Probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$ and $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$, for SIMPLE and DYNAMIC models, respectively and $x_t = a + bx_{t-1} + \nu_t$. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1)$ for heteroskedasticity adjusted models while $h_t^2 = 1$ for unadjusted ones. For all models, ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) LM_1 and LM_2 is based on (5.27) and (5.32), respectively. iii) Rejection proportions under the null of $\gamma \neq 0$ are reported. iv) Results are based on 1,000 simulations.

Table 5.9: SIMULATIONS: POWER OF LAGRANGE MULTIPLIER (LM) TEST FOR ARCH EFFECTS IN BINARY CHOICE MODELS- REJECTION FRACTIONS WHEN $h_t^2 \sim GARCH(1,1)$

		HETERO-PROBIT							
		LM_1				LM_2			
	T	100	250	500	1000	100	250	500	1000
γ	δ								
0.1	0.1	0.187	0.141	0.112	0.096	0.042	0.060	0.083	0.113
0.1	0.2	0.186	0.146	0.108	0.083	0.045	0.063	0.075	0.124
0.1	0.3	0.193	0.161	0.106	0.096	0.042	0.070	0.092	0.131
0.1	0.4	0.214	0.145	0.109	0.089	0.036	0.067	0.094	0.120
0.1	0.5	0.218	0.149	0.101	0.089	0.035	0.060	0.088	0.113
0.1	0.6	0.189	0.147	0.107	0.087	0.031	0.061	0.090	0.109
0.1	0.7	0.220	0.130	0.101	0.066	0.047	0.062	0.087	0.124
0.1	0.8	0.207	0.162	0.097	0.100	0.058	0.059	0.102	0.133
0.2	0.1	0.206	0.147	0.106	0.086	0.049	0.119	0.121	0.204
0.2	0.2	0.206	0.138	0.095	0.109	0.057	0.096	0.131	0.211
0.2	0.3	0.208	0.141	0.096	0.098	0.048	0.086	0.120	0.218
0.2	0.4	0.191	0.137	0.082	0.091	0.053	0.097	0.124	0.247
0.2	0.5	0.187	0.141	0.098	0.102	0.054	0.110	0.153	0.248
0.2	0.6	0.205	0.143	0.095	0.101	0.047	0.094	0.151	0.258
0.2	0.7	0.204	0.150	0.123	0.089	0.045	0.128	0.188	0.291
0.3	0.1	0.210	0.141	0.095	0.107	0.058	0.123	0.203	0.323
0.3	0.2	0.175	0.126	0.100	0.122	0.060	0.124	0.192	0.316
0.3	0.3	0.191	0.120	0.095	0.110	0.070	0.129	0.196	0.330
0.3	0.4	0.203	0.124	0.114	0.127	0.077	0.117	0.220	0.329
0.3	0.5	0.210	0.140	0.093	0.129	0.088	0.148	0.243	0.382
0.3	0.6	0.234	0.171	0.133	0.168	0.058	0.178	0.265	0.449
0.4	0.1	0.188	0.154	0.107	0.131	0.075	0.177	0.254	0.430
0.4	0.2	0.206	0.141	0.108	0.128	0.095	0.187	0.256	0.438
0.4	0.3	0.216	0.161	0.108	0.145	0.076	0.204	0.264	0.449
0.4	0.4	0.245	0.133	0.140	0.186	0.091	0.180	0.337	0.489
0.4	0.5	0.257	0.186	0.160	0.242	0.115	0.224	0.381	0.588
0.5	0.1	0.227	0.135	0.122	0.182	0.097	0.192	0.314	0.509
0.5	0.2	0.215	0.159	0.154	0.189	0.099	0.219	0.351	0.558
0.5	0.3	0.241	0.184	0.158	0.214	0.107	0.237	0.392	0.565
0.5	0.4	0.304	0.229	0.254	0.300	0.111	0.261	0.454	0.677
0.6	0.1	0.249	0.185	0.160	0.228	0.131	0.249	0.400	0.619
0.6	0.2	0.271	0.200	0.188	0.270	0.120	0.266	0.418	0.639
0.6	0.3	0.331	0.303	0.271	0.421	0.122	0.314	0.498	0.722
0.7	0.1	0.298	0.219	0.225	0.313	0.149	0.304	0.480	0.706
0.7	0.2	0.368	0.352	0.367	0.493	0.137	0.352	0.582	0.799
0.8	0.1	0.422	0.404	0.437	0.627	0.149	0.401	0.652	0.861
0.9	0	0.522	0.479	0.537	0.679	0.176	0.448	0.660	0.882

Notes: i) Probit Model: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_t$, for SIMPLE and $\pi_t = \alpha + \beta x_t + \xi y_{t-1}$, for DYNAMIC models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ and $x_t = a + bx_{t-1} + \nu_t$. For all models ε_t & $\nu_t \sim N(0,1)$. u_t is specified in equation (5.18) section 5.2.2. ii) LM_1 and LM_2 is based on (5.27) and (5.32), respectively. iii) Rejection proportions at 5% under the alternative of $\gamma, \delta \neq 0$ are reported. iv) Results are based on 1,000 simulations.

Based on the size and power results of LM tests, we can conclude that LM_2 performs better both under the null and the alternative. Although LM_1 test displays better performance under the ARCH alternatives, it performs poorly under more encompassing GARCH alternative. One reason LM_1 shows better power properties under the ARCH alternatives could be due to its tendency of over-rejection as observed under the null hypothesis. Furthermore, since it is most likely that model residuals from BCMs follow GARCH rather than ARCH process, better performance of LM_2 makes it a natural testing choice. Therefore, overall analysis leads to conclusion that LM_2 may be preferred to LM_1test.

5.4 Empirical Application

In this section, we provide empirical applications of the proposed hetero-probit model. There has been applications of probit model, both static as well dynamic, for prediction of business cycles (e.g. Estrella and Hardouvelis, 1991b; Estrella and Mishkin, 1997, 1998; Dueker, 1997; Kauppi and Saikkonen, 2008), interest rate changes (e.g. Eichengreen et al. (1985), Davutyan and Parke (1995); Frankle and Rose (1996), for getting early warning signals (EWS) for currency and banking crises (e.g. Eichengreen, Rose and Wyplosz (1995), Goldfajn and Valdés (1998); Demirgüç-Kunt and Detragiache (2000); Bussière and Fratzscher (2006); Candelon, Dumitrescu and Hurlin (2012), for predicting direction or bear conditions on the stock market (e.g., Chen, 2009; Nyberg, 2011; Candelon, Ahmed and Straetmans, 2012) etc. We choose to apply our model for predicting the business cycles and the stock market bears in the USA. The sample period considered is 1955Q4 to 2011Q4. We start our sample from 1955Q4 just to replicate the in-sample business cycle prediction results of Kauppi and Saikkonen (2008) over their respective sample, i.e., 1955Q4-2005Q4. We then extend the sample to 2011Q4. The estimates for prediction of bear conditions are however over the full sample of 1955Q4-2011Q4. For business cycles, we use the business cycle expansion and contraction dates provided by the National Bureau of Economic Research (NBER). We, however, extract the stock market bulls and bears via the Bry and Boschan (1971) method from the S&P 500 index. This nonparametric pattern recognition algorithm has been widely used in the literature for this purpose. Notable applications are Edwards et al. (2003a); Pagan and Sossounov (2003); Candelon et al. (2008a) and Chen (2009). The algorithm relies on some censoring criteria with respect to phase and complete cycle lengths and we follow Pagan and Sossounov (2003) in this respect. As far the explanatory variables, there is large literature which finds that the slope of yields on government's short term security and the long term bond is the best forward looking indicator for predicting the economic conditions (see e.g., Estrella and Mishkin, 1997; Dueker, 1997; Chauvet and Potter, 2001; Kauppi and Saikkonen, 2008, etc.). Therefore, we also use the term spread between the 3-month US treasury bill and the 10-year bond as an explanatory variable for both types of applications. The aforecited studies also find that the spread is useful to forecast four quarters ahead. Therefore, we use the fourth lag of the yield spread. Lastly, we estimate models with and without heteroskedastic adjustment using both the static as well as the dynamic versions of probit model.

Table 5.10, which is split into three panels, reports the results for prediction of both business and financial cycles. Panel A contains results for business cycles over the sample period studied in Kauppi and Saikkonen (2008), i.e., 1955Q4-2005Q4. In this and the subsequent panels, we report the parameters estimates for both mean and variance, their respective standard errors (in parenthesis), the pseudo-R^2 proposed in Estrella (1998), the value of log-likelihood, the mean squared forecast error (MSFE) and the $p-$values for the LM tests of no ARCH effects at lags 1. Comparing the results from simple probit with its heteroskedasticity adjusted version, the GARCH-probit, a clear gain in explanatory power is quite obvious as the R^2 improves from 25% to 34%. The log-likelihood ratio at 16.00 is too large to be insignificant. At the same time, the MSFE is smaller for the heteroskedasticity augmented model. Furthermore, a large and significant ARCH coefficient ($\widehat{\gamma}$) clearly points to the importance of short term shocks for the business cycles. However, the null of no heteroskedasticity is rejected for both models. One reason for this could be that the adjusted model might still be potentially misspecified in having a lower order GARCH process. Therefore, increasing its lag order may further improve the fit. Additionally, since the periods of downturns and upturns exhibit clustering and persistence, adding the lags of dependent variable might help. Therefore, we also add the first lag of the indicator variable as an explanatory variable. Not only is there improvement in all respects, the null of homoskedasticity now cannot be rejected. The lagged dummy variable takes care of the short term shocks as the ARCH parameter drops in magnitude and becomes insignificance. Even the long run effects, as measured by the GARCH parameter ($\widehat{\delta}$), turns out to be insignificant. It is therefore, clear that the model needs to be adjusted for heteroskedasticity.

Table 5.10: Empirics: Prediction of the U.S. Business and Financial Cycles via probit Models

	Estimates					Diagnostics			
	$\widehat{\gamma}$	$\widehat{\delta}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\xi}$	R^2	LLK	RMSE	LM(1)
Panel A: Business Cycles - 1955Q4-2011Q4									
Simple			**-0.232**	**-0.751**		0.257	-67.1	0.106	***0.001***
			(0.013)	(0.012)					
Hetero	**0.988**	0.011	**-0.020**	**-0.032**		0.341	-59.1	0.100	***0.000***
	(0.053)	(0.053)	(0.001)	(0.002)					
Dynamic			**-1.130**	**-0.419**	**1.952**	0.475	-46.2	0.068	*0.093*
			(0.018)	(0.010)	(0.024)				
Dyn-Hetero	0.264	0.091	**-0.586**	**-2.528**	**1.429**	-0.220	-114.0	0.086	***0.035***
	(0.246)	(0.413)	(0.188)	(0.852)	(0.176)				
Panel B: Business Cycles - 1955Q4-2011Q4									
Simple			**-0.199**	**-0.674**		0.231	-82.0	0.116	***0.000***
			(0.011)	(0.009)					
Hetero	**0.999**	0.000	**-0.019**	**-0.029**		0.349	-69.1	0.105	***0.000***
	(0.040)	(0.040)	(0.001)	(0.001)					
Dynamic			**-1.129**	**-0.416**	**2.091**	0.507	-52.0	0.068	*0.101*
			(0.016)	(0.009)	(0.020)				
Dyn-Hetero	**0.069**	**0.874**	**-0.784**	**-0.324**	**1.516**	0.509	-51.7	0.069	***0.012***
	(0.015)	(0.058)	(0.027)	(0.009)	(0.038)				
Panel C: Financial Cycles - 1955Q4-2011Q4									
Simple			**-0.270**	**-0.152**		0.020	-135.7	0.212	***0.000***
			(0.009)	(0.005)					
Hetero	**0.998**	0.000	**-0.051**	**-0.004**		0.199	-115.6	0.179	***0.000***
	(0.036)	(0.036)	(0.002)	(0.000)					
Dynamic			**-1.422**	**0.011**	**2.314**	0.528	-75.6	0.098	***0.044***
			(0.014)	(0.006)	(0.016)				
Dyn-Hetero	0.000	0.288	**-1.425**	**0.012**	**2.306**	0.530	-75.4	0.098	***0.045***
	(0.009)	(343)	(0.016)	(0.006)	(0.017)				

Notes: i) Probit Models: $Pr(y_t = 1) = \Phi(\pi_t/h_t)$, where $\pi_t = \alpha + \beta x_{t-4}$ for Simple, $\pi_t = \alpha + \beta x_{t-4} + \xi y_{t-1}$ for Dynamic models. $h_t^2 = 1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)$ for heteroskedasticty adjusted model while $h_t = 1$ for unadjusted ones. ii) Standard errors in (parenthesis) below the estimates. iii) R^2 is the Pseudo-R^2. iv) FMSE = Forecast Mean Squared Error. v) $LM(q)$ is the test of no heteroskedasticity (No ARCH effect) at lag q, based on (5.32). $p-$values are reported.

Next we increase the sample size to 2011Q4, which includes the periods of high volatility due the recent sub-prime crises, and re-estimate the model. Panel B of Table 5.10 reports the results. All the relevant metrics tell a story quite similar to the previous one. However, while the explanatory power of the hetero-probit remains the same, that of static model deteriorates (from 26% to 23%). As the extended sample contains periods of high volatility, the loss of explanatory power implies that the simple model is incapable of incorporating and reflecting the dynamically changing behavior of the data. Furthermore, while the two variance parameters were insignificant in the dynamic-hetero probit over 1955Q4-2005Q4 sample period, both now turn out to be significant and persistent ($\gamma + \delta = 0.943$).

Our next empirical application is for the financial market data, which is typically characterized by heteroskedasticity and persistence. Specifically, we predict the bear conditions on the US stock market. We locate the peaks and troughs in the S&P 500 index from 1955Q4 to 2011Q4 via Bry and Boschan (1971) algorithm using the censoring criteria of Pagan and Sossounov (2003) and Candelon et al. (2008b). We then term periods from peak to trough as bears (= 1) and the periods from trough to peak as bulls (= 0)[9]. We again use fourth lag of term spread as the only explanatory variable because of its forward looking behavior. The results are reported in panel C of Table 5.10. Looking at the unadjusted static model, it can be seen that although the spread is significant, and hence relevant for bear market prediction, the model explains hardly 2% of variation in bear conditions. The GARCH-probit on the other hand explains around 20% of the variation with the same explanatory variable, which is a substantial gain. The latter is also accompanied by a lower MSFE and a significant value addition in terms of likelihood ratio. However, as the bear conditions also tend to be persistent, we account for this by adding the lagged bear dummy to the information set. The improvement is now even higher. However, in all instances, the LM test for no heteroskedasticity does not yield favorable results. Perhaps the level of heteroskedasticity in the financial time series is deeper than what could be captured by a GARCH(1,1) adjustment.

Overall, the simulation as well as empirical results show that there indeed is the need for heteroskedastic adjustment in the binary choice models for the macro-financial time series applications. The adjustment leads to substantial improvement over the unadjusted models in terms of fit and MSE. Given that the binary choice models are used for predicting economic and financial cycles and as early warning tools, the value addition gained by the heteroskedastic adjustment suggests that it will lead to better and consistent policy and investment decisions. The model can be used for tactical asset allocation and active portfolio re-balancing.

5.5 Conclusion

The macro-financial data are characterized by serial dependence and volatility clustering, leading to conditional heteroskedasticity. The binary choice models (BCM) have been extensively used for business and financial cycles prediction, as the early warning tools for currency crisis etc. Since the foregoing applications involve macro-financial variables, the model errors are likely to be condi-

[9]See Chapter 1, section 2.2.2 for further details.

tionally heteroskedastic. This will in turn lead to inconsistent parameter estimates. To guard against this, it is proposed to conditionally adjust the BCMs for heteroskedasticity via GARCH(p, q) process of Bollerslev (1986). Mindful of the fact that the long run variance of BCMs is frozen to unity to achieve identification, we propose to let the variance fluctuate in the short run, while leaving it frozen to unity in the long run. The simulation results show that such an adjustment dampens the bias level of estimates and leads to efficient estimates in mean squared error sense. We also propose Lagrange Multiplier tests for testing the ARCH effects in the model residuals. The simulation results on the size and power of LM_1 and LM_2 tests show that latter test performs better. The LM_1 test however shows better power for small samples under ARCH alternatives. This advantage however dilutes when a GARCH alternative is considered. Empirical applications of the proposed model to business and financial cycles predictions reveal that the heteroskedasticity augmentation improves the model fit and yields a lower mean squared forecast error. Post adjustment, it is also observed that ARCH effects are more pronounced, implying the significant role played by persistence of short term shocks. However, taking care of the persistence of cyclical periods via addition of lagged dummy variable not only substantially improves the fit but the relative magnitude of ARCH effects also become insignificant. While after the latter adjustment for persistence we could not to reject the null hypothesis of no heteroskedasticity for business cycles, we fail to do so for bear market predictions. This might imply that a higher order GARCH process may be desirable to capture the remaining conditional heteroskedasticity in financial time series.

5.6 Appendix

The Model

Consider the model relating an unobserved latent variable y_t^* with a row vector of exogenous variables, $\boldsymbol{X_t}$,

$$y_t^* = \boldsymbol{X_t}\boldsymbol{\beta} + u_t \tag{5.42}$$

Together with assumptions (5.6), (5.7) and (5.10), the conditional log-likelihood is given by

$$\mathcal{L} = \sum_{t=1}^{T} \left(y_t \log \Phi(\pi_t) + (1 - y_t) \log[1 - \Phi(\pi_t)] \right), \tag{5.43}$$

where $\Phi(.)$ is the standard normal distribution and $\pi_t = \boldsymbol{X_t}\boldsymbol{\beta}/h_t$.

Utilizing constraints (5.11), the first two conditional moments are given by

$$E[y_t|\varphi_{t-1}] = \Phi(\pi_t) \tag{5.44}$$

and

$$\begin{aligned} E[(y_t - E[y_t])^2|\varphi_{t-1}] &= E\left[y_t - 2y_t\Phi(\pi_t) + \Phi(\pi_t)^2|\varphi_{t-1} \right], \\ &= \Phi(\pi_t)(1 - \Phi(\pi_t)) \end{aligned} \tag{5.45}$$

Finally, for notational convenience, let $h_t = \sqrt{1 + \gamma(u_{t-1}^2 - 1) + \delta(h_{t-1}^2 - 1)} = \sqrt{1 + \boldsymbol{Z_t}\boldsymbol{\Gamma}}$, where $\boldsymbol{Z_t} = [u_{t-1}^2 \;\; h_{t-1}^2]$ is a row vector and $\boldsymbol{\Gamma} = (\gamma, \delta)$ is a column vector of parameters.

The Score

First, partially differentiating (5.43) w.r.t. $\boldsymbol{\beta}$ yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \boldsymbol{\beta}} &= \frac{\boldsymbol{X_t} y_t \phi(\pi_t)}{\sqrt{\boldsymbol{Z_t}\boldsymbol{\Gamma} + 1}\Phi(\pi_t)} - \frac{\boldsymbol{X_t}(1 - y_t)\phi(\pi_t)}{\sqrt{\boldsymbol{Z_t}\boldsymbol{\Gamma} + 1}\,(1 - \Phi(\pi_t))} \\ &= \frac{\boldsymbol{X_t}\phi(\pi_t)\,(\Phi(\pi_t) - y_t)}{\sqrt{\boldsymbol{Z_t}\boldsymbol{\Gamma} + 1}\,(\Phi(\pi_t) - 1)\,\Phi(\pi_t)} \\ &= \left[\frac{\phi(\pi_t)\,(y_t - \Phi(\pi_t))}{h_t \times (1 - \Phi(\pi_t))\,\Phi(\pi_t)} \right] \boldsymbol{X_t}. \end{aligned} \tag{5.46}$$

And taking partial derivative w.r.t. $\boldsymbol{\Gamma}$ we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mathbf{\Gamma}} &= \frac{(\boldsymbol{X_t\beta})\boldsymbol{Z_t}(1-y_t)\phi(\pi_t)}{2(\boldsymbol{Z_t}\mathbf{\Gamma}+1)^{3/2}\,(1-\Phi(\pi_t))} - \frac{(\boldsymbol{X_t\beta})\boldsymbol{Z_t}y_t\phi(\pi_t)}{2(\boldsymbol{Z_t}\mathbf{\Gamma}+1)^{3/2}\Phi(\pi_t)} \\
&= \frac{(\boldsymbol{X_t\beta})\boldsymbol{Z_t}\phi(\pi_t)\,(y_t-\Phi(\pi_t))}{2(\boldsymbol{Z_t}\mathbf{\Gamma}+1)^{3/2}\,(\Phi(\pi_t)-1)\,\Phi(\pi_t)} \\
&= -\left[\frac{\phi(\pi_t)\,(y_t-\Phi(\pi_t))}{h_t\times(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\frac{1}{2h_t^2}(\boldsymbol{X_t\beta})\boldsymbol{Z_t}.
\end{aligned}
\tag{5.47}
$$

Now, collecting the terms in (5.46) and (5.47), the score is given by

$$
\left[\frac{\phi(\pi_t)\,(y_t-\Phi(\pi_t))}{h_t\times(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\left[-\frac{1}{2h_t^2}(\boldsymbol{X_t\beta})\boldsymbol{Z_t} \qquad \boldsymbol{X_t}\right]. \tag{5.48}
$$

The score under the null hypothesis that $\gamma=\delta=0$ is

$$
\left[\frac{\phi(\pi_t)\,(y_t-\Phi(\pi_t))}{(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\left[-\frac{(\boldsymbol{X_t\beta})\boldsymbol{Z_t}}{2} \qquad \boldsymbol{X_t}\right]. \tag{5.49}
$$

The Expectations of Hessian

For the Hessian, first partially differentiate (5.46) w.r.t. $\boldsymbol{\beta}$, to get

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}'} &= -\frac{\boldsymbol{X_t'X_t}(1-y_t)\phi(\pi_t)}{(\mathbf{\Gamma}\boldsymbol{Z_t}+1)\,(1-\Phi(\pi_t))} + \frac{\boldsymbol{X_t'X_t}y_t\phi(\pi_t)}{(\boldsymbol{Z_t}\mathbf{\Gamma}+1)\Phi(\pi_t)} \\
&\quad - \frac{\boldsymbol{X_t'X_t}(1-y_t)\phi(\pi_t)^2}{(\boldsymbol{Z_t}\mathbf{\Gamma}+1)\,(1-\Phi(\pi_t))^2} - \frac{\boldsymbol{X_t'X_t}y_t\phi(\pi_t)^2}{(\boldsymbol{Z_t}\mathbf{\Gamma}+1)\Phi(\pi_t)^2} \\
&= \underbrace{\frac{\boldsymbol{X_t'X_t}\left((\Phi(\pi_t)-1)\,\Phi(\pi_t)\,(\Phi(\pi_t)-y_t)\,\phi(\pi_t)\right)}{(\boldsymbol{Z_t}\mathbf{\Gamma}+1)\,(\Phi(\pi_t)-1)^2\,\Phi(\pi_t)^2}}_{A} \\
&\quad + \underbrace{\frac{\boldsymbol{X_t'X_t}\left(\phi(\pi_t)^2\left(2y_t\Phi(\pi_t)-\Phi(\pi_t)^2-y_t\right)\right)}{(\boldsymbol{Z_t}\mathbf{\Gamma}+1)\,(\Phi(\pi_t)-1)^2\,\Phi(\pi_t)^2}}_{B}.
\end{aligned}
\tag{5.50}
$$

Taking expectations and using (5.44) and (5.45), part 'A' in (5.50) vanishes and part 'B' reduces to

$$
E[\mathcal{I}_{\beta\beta}] = E\left[\frac{\partial\mathcal{L}}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}'}\right] = -\left[\frac{\phi(\pi_t)^2}{h_t^2\times(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\boldsymbol{X_t'X_t}. \tag{5.51}
$$

Second, take partial derivative of (5.47) w.r.t. $\mathbf{\Gamma}$,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \boldsymbol{\Gamma} \partial \boldsymbol{\Gamma}'} &= -\frac{(\boldsymbol{X_t\beta})^2 \boldsymbol{Z_t'Z_t}(1-y_t)\phi(\pi_t)}{4(\boldsymbol{Z_t\Gamma}+1)^3\left(1-\Phi(\pi_t)\right)} + \frac{(\boldsymbol{X_t\beta})^2 \boldsymbol{Z_t'Z_t} y_t \phi(\pi_t)}{4(\boldsymbol{Z_t\Gamma}+1)^3 \Phi(\pi_t)} \\
&\quad - \frac{(\boldsymbol{X_t\beta})^2 \boldsymbol{Z_t'Z_t}(1-y_t)\phi(\pi_t)^2}{4(\boldsymbol{Z_t\Gamma}+1)^3\left(1-\Phi(\pi_t)\right)^2} - \frac{(\boldsymbol{X_t\beta})^2 \boldsymbol{Z_t'Z_t} y_t \phi(\pi_t)^2}{4(\boldsymbol{Z_t\Gamma}+1)^3 \Phi(\pi_t)^2} \\
&\quad - \frac{3(\boldsymbol{X_t\beta}) \boldsymbol{Z_t'Z_t}(1-y_t)\phi(\pi_t)}{4(\boldsymbol{Z_t\Gamma}+1)^{5/2}\left(1-\Phi(\pi_t)\right)} + \frac{3(\boldsymbol{X_t\beta}) \boldsymbol{Z_t'Z_t} y_t \phi(\pi_t)}{4(\boldsymbol{\Gamma Z_t}+1)^{5/2} \Phi(\pi_t)} \\
&= -\underbrace{\frac{(\boldsymbol{X_t\beta}) \boldsymbol{Z_t'Z_t}\left(\boldsymbol{X_t\beta}\sqrt{\boldsymbol{Z_t\Gamma}+1}\left(\Phi(\pi_t)-1\right)\Phi(\pi_t)\left(y_t-\Phi(\pi_t)\right)\phi(\pi_t)\right)}{4(\boldsymbol{Z_t\Gamma}+1)^{7/2}\left(\Phi(\pi_t)-1\right)^2\Phi(\pi_t)^2}}_{A} \\
&\quad + \underbrace{\frac{(\boldsymbol{X_t\beta}) \boldsymbol{Z_t'Z_t}\left(\boldsymbol{X_t\beta}\sqrt{\boldsymbol{Z_t\Gamma}+1}\phi(\pi_t)^2\left(-2y_t\Phi(\pi_t)+\Phi(\pi_t)^2+y_t\right)\right)}{4(\boldsymbol{Z_t\Gamma}+1)^{7/2}\left(\Phi(\pi_t)-1\right)^2\Phi(\pi_t)^2}}_{B} \\
&\quad + \underbrace{\frac{(\boldsymbol{X_t\beta}) \boldsymbol{Z_t'Z_t}\left(3(\boldsymbol{Z_t\Gamma}+1)\left(\Phi(\pi_t)-1\right)\Phi(\pi_t)\phi(\pi_t)\left(y_t-\Phi(\pi_t)\right)\right)}{4(\boldsymbol{Z_t\Gamma}+1)^{7/2}\left(\Phi(\pi_t)-1\right)^2\Phi(\pi_t)^2}}_{A}
\end{aligned}
\tag{5.52}
$$

Again, using (5.44) and (5.45), parts labeled 'A' in (5.52) vanish and part 'B' reduces to

$$
E[\mathcal{I}_{\Gamma\Gamma}] = E\left[\frac{\partial \mathcal{L}}{\partial \boldsymbol{\Gamma}\partial \boldsymbol{\Gamma}'}\right] = -\left[\frac{\phi(\pi_t)^2}{h_t^2 \times \left(1-\Phi(\pi_t)\right)\Phi(\pi_t)}\right]\frac{1}{4h_t^4}(\boldsymbol{X_t\beta})^2 \boldsymbol{Z_t'Z_t}. \tag{5.53}
$$

Finally, partially differentiating (5.46) w.r.t. $\boldsymbol{\Gamma}$, we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \beta \partial \boldsymbol{\Gamma}} &= \frac{(\boldsymbol{X_t\beta}) \boldsymbol{X_t'Z_t}(1-y_t)\phi(\pi_t)}{2(\boldsymbol{\Gamma Z_t}+1)^2\left(1-\Phi(\pi_t)\right)} - \frac{(\boldsymbol{X_t\beta}) \boldsymbol{X_t'Z_t} y_t \phi(\pi_t)}{2(\boldsymbol{\Gamma Z_t}+1)^2 \Phi(\pi_t)} \\
&\quad + \frac{(\boldsymbol{X_t\beta}) \boldsymbol{X_t'Z_t}(1-y_t)\phi(\pi_t)^2}{2(\boldsymbol{\Gamma Z_t}+1)^2\left(1-\Phi(\pi_t)\right)^2} + \frac{(\boldsymbol{X_t\beta}) \boldsymbol{X_t'Z_t} y_t \phi(\pi_t)^2}{2(\boldsymbol{\Gamma Z_t}+1)^2 \Phi(\pi_t)^2} \\
&\quad + \frac{\boldsymbol{X_t'Z_t}(1-y_t)\phi(\pi_t)}{2(\boldsymbol{\Gamma Z_t}+1)^{3/2}\left(1-\Phi(\pi_t)\right)} - \frac{\boldsymbol{X_t'Z_t} y_t \phi(\pi_t)}{2(\boldsymbol{\Gamma Z_t}+1)^{3/2} \Phi(\pi_t)}
\end{aligned}
\tag{5.54}
$$

$$
\begin{aligned}
&= \underbrace{\frac{\boldsymbol{X_t'Z_t}\left((\boldsymbol{X_t\beta})\sqrt{\boldsymbol{\Gamma Z_t}+1}\,(\Phi(\pi_t)-1)\,\Phi(\pi_t)\,(y_t-\Phi(\pi_t))\,\phi(\pi_t)\right)}{2(\boldsymbol{Z_t}\boldsymbol{\Gamma}+1)^{5/2}\,(\Phi(\pi_t)-1)^2\,\Phi(\pi_t)^2}}_{A} \\
&+ \underbrace{\frac{\boldsymbol{X_t'Z_t}\left((\boldsymbol{X_t\beta})\sqrt{\boldsymbol{Z_t\Gamma}+1}\phi(\pi_t)^2\left(-2y_t\Phi(\pi_t)+\Phi(\pi_t)^2+y_t\right)\right)}{2(\boldsymbol{Z_t}\boldsymbol{\Gamma}+1)^{5/2}\,(\Phi(\pi_t)-1)^2\,\Phi(\pi_t)^2}}_{B} \qquad (5.55) \\
&+ \underbrace{\frac{\boldsymbol{X_t'Z_t}\left((\boldsymbol{Z_t}\boldsymbol{\Gamma}+1)\,(\Phi(\pi_t)-1)\,\Phi(\pi_t)\phi(\pi_t)\,(y_t-\Phi(\pi_t))\right)}{2(\boldsymbol{Z_t}\boldsymbol{\Gamma}+1)^{5/2}\,(\Phi(\pi_t)-1)^2\,\Phi(\pi_t)^2}}_{A}
\end{aligned}
$$

Making use of (5.44) reduces part ‘A’ in (5.55) to zeros and using (5.45) leads to

$$
\begin{aligned}
E[\mathcal{I}_{\odot -}] = E\left[\frac{\partial \mathcal{L}}{\partial \beta \partial \boldsymbol{\Gamma}'}\right] &= -\frac{((\boldsymbol{X_t\beta})\boldsymbol{X_t'Z_t}\phi(\pi_t)^2}{2(\boldsymbol{Z_t}\boldsymbol{\Gamma}+1)^2\,(\Phi(\pi_t)-1)\,\Phi(\pi_t)} \\
&= \left[\frac{\phi(\pi_t)^2}{h_t^2\times(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\frac{1}{2h_t^2}(\boldsymbol{X_t\beta})\boldsymbol{X_t'Z_t}.
\end{aligned} \qquad (5.56)
$$

Consequently, the variance-covariance based on the expectations of Information matrix is

$$
\mathbf{V}(\boldsymbol{\Theta}) = E[\mathcal{I}(\boldsymbol{\Theta})|\varphi_{t-1}] = -E\left[\frac{\partial^2 \mathcal{L}}{\partial \boldsymbol{\Theta} \partial \boldsymbol{\Theta}'}\right] = -\begin{bmatrix} \mathcal{I}_{\beta\beta} & \mathcal{I}_{\beta\Gamma} \\ \mathcal{I}_{\beta\Gamma}' & \mathcal{I}_{\Gamma\Gamma} \end{bmatrix} \qquad (5.57)
$$

where $\boldsymbol{\Theta} = (\boldsymbol{\Gamma'}, \boldsymbol{\beta'})$.

The expectations of Hessian under the null hypothesis of $\gamma = \delta = 0$ indeed does not contain h_t, i.e.,

$$
\begin{aligned}
\mathcal{I}_{\beta\beta} &= E\left[\frac{\partial^2 \mathcal{L}}{\partial \beta \partial \beta'}\right] = \sum_{t=1}^{T}\left[-\frac{[\phi(\pi_t)]^2}{(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\boldsymbol{X_t'X_t}, \\
\mathcal{I}_{\Gamma\Gamma} &= E\left[\frac{\partial^2 \mathcal{L}}{\partial \Gamma \partial \Gamma'}\right] = \sum_{t=1}^{T}\left[-\frac{[\phi(\pi_t)]^2}{(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\frac{1}{4}(\boldsymbol{X_t\beta})^2\boldsymbol{Z_t'Z_t},
\end{aligned} \qquad (5.58)
$$

and

$$
\mathcal{I}_{\Gamma\beta} = E\left[\frac{\partial^2 \mathcal{L}}{\partial \Gamma \partial \beta'}\right] = \sum_{t=1}^{T}\left[\frac{[\phi(\pi_t)]^2}{(1-\Phi(\pi_t))\,\Phi(\pi_t)}\right]\frac{1}{2}(\boldsymbol{X_t\beta})\boldsymbol{X_t'Z_t}.
$$

Chapter 6

Conclusion

This thesis consists of four self-contained essays on business and financial cycles. We venture to predict the business and financial cycles, accounting for their persistence, the clustering effects and heteroskedasticity using the dynamic extensions of binary choice models. Particularly, in Chapter 2, we forecast the bear phases on the U.S. stock market. We find that while forecasting these cycles it pays off to account for the stylized facts of the cyclical series. Whereas the prediction of equity market cycles (bears or bulls) might help regulators and policy makers take proactive measures to contain the excesses in the asset prices, it turns out that it is equally valuable for the investors to time the market and engage in active trading and beat the passive buy-and-hold strategies.

Focusing on another financial asset class, in Chapter 3 we consider the cycles in the bilateral exchange rates. Here we study cycles in world's six major currencies which are considered as either 'reserve currencies' or 'funding currencies' or 'investment currencies'. We extract cyclical periods of appreciation and depreciation non-parametrically and try to forecast these episodes one to twenty four months ahead using five risk factors. Broadly, these risk factors include violations of UIP, RPPP, pseudo-parity with equity prices, liquidity pressures and term spread. With these risk factors, exchange rate indeed exhibits cyclical variations as the periods of appreciation follow the periods of depreciation, and vice versa. This implies that the policy makers can utilize these signals to smoothen the currency misalignment and restore its competitiveness. Further, as the carry trade is a major trading activity on the forex market, any adverse movements in the target or funding currencies may erode traders' cumulative earnings. Therefore, foretasted periods of appreciation and depreciation may also help carry traders avoid possible losses by portfolio rebalancing.

Penultimately, in Chapter 4, we study the bilateral real and financial cycle synchronization within nine eurozone countries. Using the probit framework, we find strong cross-country synchronization in both business cycles and financial cycles. This is reflected by statistically and economically significant marginal effects within the probit framework. Moreover, financial synchronization dominates business cycle synchronization in the eurozone, especially after the introduction of the single currency: whereas the euro sample coincides with a strong increase in financial synchronization, business cycle synchronization does not change much. For some country pairs, we even find some evidence of "de-coupling" business cycles but the majority of marginal business cycle effects do not change much over time. Controlling for endogeneity does not fundamentally alter our results. Our results suggest that monetary integration has brought more financial integration; but the impact of monetary integration on business cycle synchronization remains limited or even seems to lead to a "de-coupling" of peripheral countries' business cycles relative to the core countries in a number of cases. The former observation supports the often heard plea for more international macro-prudential regulation whereas the latter observation gives ammunition to those economists that always stressed that the euro zone architecture is unfinished business and that the conditions for an optimum currency area are not fulfilled.

Finally, in Chapter 5, we take up hitherto neglected aspect of conditional heteroskedasticity in binary modeling framework. We build on the observation that the serial dependence and volatility clustering, inter alia, lead to conditional heteroskedasticity in the model disturbances (Engle, 1982) for which we propose an adjustment along the lines of Engle (1982) and Bollerslev (1986). Specifically, we propose a GARCH-type adjustment for the conditional variance of model errors in the short run, while leaving the unconditional variance fixed to unity to achieve identification. The results based on the simulations and empirical applications to predicting the US business and financial cycles confirm the utility of heteroskedastic adjustment. Moreover, we also propose LM type tests for ARCH effects in BCMs. Of the proposed LM tests one based on the expected Information matrix exhibits smaller size distortions and higher power properties.

Bibliography

Acharya, V. V. and Pedersen, L. H. (2005). Asset pricing with liquidity risk, *Journal of Financial Economics* **77**(2): 375–410.

Adjaoute, K. and Danthine, J.-P. (2004). Equity returns and integration: Is Europe changing, *Oxford Review of Economic Policy* **20**(4): 555–570.

Aggarwal, R. (1981). Exchange rates and stock prices: A study of the us capital markets under floating exchange rates, *Akron Business and Economic Review* **12**(4): 7–12.

Albert, J. and Chib, S. (1993). Bayesian Analysis of Binary and Ploychotomous Response Data, *Journal of the American Statistical Association* **88**: 669–679.

Amemiya, T. (1985). *Advanced Econometrics*, Harvard University Press.

Anatolyev, S. and Gospordinov, N. (2010). Modeling Financial Return Dynamics via Decomposition, *Journal of Business and Economic Statistics* **28(2)**: 232–245.

Artis, J. M. (2003). Reflections on the optimal currency area (OCA) criteria in the light of EMU, *International Journal of Finance and Economics* **8**(4): 297–307.

Artis, J. M. and Zhang, W. (1997). International business cycles and the ERM: Is there a European business cycle, *International Journal of Finance and Economics* **2**(1): 1–16.

Artis, J. M. and Zhang, W. (1999). Further evidence on the international business cycle and the ERM: Is there a European business cycle, *Oxford Economic Papers* **51**(1): 120–132.

Artis, M., Krolzig, H.-M. and Toro, J. (2004). The European business cycle, *Oxford Economic Papers* **56**(1): 1–44.

Bahmani-Oskooee, M. and Sohrabian, A. (1992). Stock prices and the effective exchange rate of the dollar, *Applied economics* **24**(4): 459–464.

Balduzzi, P. and Lynch, A. W. (1999). Transaction Costs and predictability: some utility cost calculations, *Journal of Financial Economics* **52**: 47–78.

Baxter, M. and King, R. G. (1999). Measuring business cycles: Approximate bandpass filters for economic time series, *The Review of Economics and Statistics* **81**(4): 575–593.

Beine, M. and Candelon, B. (2011). Liberalization and stock market comovement between emerging economies, *Quantitative Finance* **11**(2): 299–312.

Bekaert, G., Harvey, R. C., Lundblad, T. C. and Siegel, S. (2010). *The European Union, the Euro, and equity market integration*, NBER Working Paper No. 16583.

Berg, A. and Pattillo, C. (1999). Predicting currency crises: The indicators approach and an alternative, *Journal of International Money and Finance* **18**(4): 561–586.

Berger, D., Chaboud, A., Chernenko, S., Howorka, E. and Wright, J. (2008). Order flow and exchange rate dynamics in electronic brokerage system data, *Journal of International Economics* **75**(1): 93–109.

Berkowitz, J. and Giorgianni, L. (2001). Long-horizon exchange rate predictability?, *Review of Economics and Statistics* **83**(1): 81–91.

Bilson, J. (1978). Rational expectations and the exchange rate, *The economics of exchange rates* pp. 75–96.

Bilson, J. (1990). *The Currency-Hedging Debate*, International Financing Review Publishing, London, chapter Technical Currency Trading.

Birchenhall, C. R., Jessen, H., Osborn, D. R. and Simpson, P. (1999). Predicting U.S. Business Cycle Regimes, *Journal of Business & Economic Statistics* **17(3)**: 313–323.

BIS (2005). Foreign exchange market intervention in emerging markets: motives, techniques and implications, *BIS Papers* **24**.

BIS (2010). Triennial Central Bank Survey Foreign exchange and derivatives market activity in April 2010, *Bank for International Settelments (BIS) Review* .

Boehmer, E., Jones, C. M. and Zhang, X. (2008). Which Shorts Are Informed?, *Journal of Finance* **63(2)**: 491–527.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics* **31**: 307–327.

Bollerslev, T. (1988). On the correlation structure for the generalized autoregressive conditional heteroskedastic process, *Journal of Time Series Analysis* **9**(2): 121–131.

Bordo, M., Eichengreen, B., Klingebiel, D. and Martinez-Peria, M. (2001). Is the crisis problem growing more severe?, *Economic policy* **16**(32): 51–82.

Borio, C. and Lowe, P. (2002). Asset Prices, Financial and Monetary Stability: Exploring the Nexus, *BIS Working Papers* **No. 114**.

Böwer, U. and Guillemineau, C. (2006). *Determinants of business cycle synchronisation across Euro Area countries*, SSRN Working Paper.

Breusch, T. S. and Pagan, A. R. (1979). A simple test for heteroscedasticity and random coefficient variation, *Econometrica: Journal of the Econometric Society* pp. 1287–1294.

Breuss, F. (2011). *Downsizing the eurozone into an OCA or entry into a fiscal transfer union*, IDEAS RePEc.

Brock, W. A. and Hommes, C. H. (1997). A rational route to randomness, *Econometrica: Journal of the Econometric Society* pp. 1059–1095.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model, *Journal of Economic dynamics and Control* **22**(8-9): 1235–1274.

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity, *Review of Financial studies* **22**(6): 2201–2238.

Brunnermeier, M., Nagel, S. and Pedersen, L. (2008). Carry Trades and Currency Crashes, *NBER Macroeconomics Annual* **23**(1): 313–348.

Bry, G. and Boschan, C. (1971). *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, Columbia University Press, National Bureau of Economic Reasearch, New York.

Burns, A. F. and Mitchell, W. C. (1946). *Measuring Business Cycles*, Book Series Studies in Business Cycles, National Bureau of Economic Resaerch (NBER), New York.

Bussière, M. and Fratzscher, M. (2006). Towards a new early warning system of financial crises, *Journal of International Money and Finance* **25**(6): 953–973.

Calzolari, G. and Fiorentini, G. (1998). A tobit model with GARCH errors, *Econometric Reviews* **17**(1): 85–104.

Campbell, J. Y., Lo, A. W. and A., M. C. (1997). *The Econometrics of Financial Markets*, Princeton University Press.

Candelon, B., Ahmed, J. and Straetmans, S. (2012). Predicting and Capitalizing on Stock Market Bears in the US, *METEOR Research Memo* **12/019**.

Candelon, B., Dumitrescu, E. and Hurlin, C. (2012). How to Evaluate an Early-Warning System: Toward a Unified Statistical Framework for Assessing Financial Crises Forecasting Methods, *IMF Economic Review* **60**(1): 75–113.

Candelon, B., Piplack, J. and Straetmans, S. (2008a). On Measuring Synchronization of Bulls and Bears: The case of East Asia, *Journal of Banking and Finance* **32**: 1022–1035.

Candelon, B., Piplack, J. and Straetmans, S. (2008b). On measuring the synchronization of bulls and nears: The case of East Asia, *Journal f Banking and Finance* **32**: 1022–1035.

Candelon, B., Piplack, J. and Straetmans, S. (2009). Multivariate business cycle synchronization in small samples, *Oxford Bulletin of Economics and Statistics* **71**(5): 715–737.

Caporale, G. M. and Spagnolo, N. (2011). Stock market integration between three CEECs, Russia, and the UK, *Review of International Economics* **19**(1): 158–169.

Cavallo, M. (1984). Interest rates, carry trades, and exchange rate movements, *FRBSF Economic Letter* (Nov 17).

Chaboud, A. P. and Wright, J. H. (2005). Uncovered interest parity: it works, but not for long, *Journal of International Economics* **66**(2): 349–362.

Chan, C. K., Gup, E. B. and Pan, M.-S. (1997). International stock market efficiency and integration: A study of eighteen nations, *Journal of Business and Accounting* **24**(6): 803–813.

Chauvet, M. and Potter, S. (2000a). Coincident and Leading Indicators of Stock Market, *Journal of Empirical Finance* **7**: 87–111.

Chauvet, M. and Potter, S. (2000b). Coincident and Leading Indicators of Stock Market, *Journal of Empirical Finance* **7**: 87–111.

Chauvet, M. and Potter, S. (2001). Forecasting recessions using yield curve, *Federal Reserve Bank of New York Staff Paper 134* .

Chen, S.-S. (2009). Predicting the bear stock market: Macroeconomic variables as Leading Indicators, *Journal f Banking and Finance* **33**: 211–223.

Chen, Y.-c. and Tsang, K. P. (2013). What Does the Yield Curve Tell Us About Exchange Rate Predictability?, *Review of Economics and Statistics* **95(1)**: 185–205.

Cheung, Y.-W. and Chinn, M. D. (2001). Currency traders and exchange rate dynamics: a survey of the us market, *Journal of International Money and Finance* **20**(4): 439 – 471.

Chinn, M. and Meese, R. (1995). Banking on currency forecasts: how predictable is change in money?, *Journal of International Economics* **38**(1): 161–178.

Christiano, L. and Fitzgerald, T. J. (2003). The bankpass filter, *International Economic Review* **44**(2): 435–465.

Christiansen, C., Ranaldo, A. and Söderlind, P. (2011). The time-varying systematic risk of carry trade strategies, *Journal of Financial and Quantitative Analysis* **46**(04): 1107–1125.

Christoffersen, P. F. and Diebold, F. X. (2006). Financial Asset Returns, Direction of Change Forecasting, and Volatility Dynamics, *Management Science* **52(8)**: 1273–1287.

Clarida, R. H. and Waldman, D. (2008). Is Bad News about Inflation Good News for the Exchange Rate? And, If So, Can That Tell Us Anything about the Conduct of Monetary Policy?, *Asset Prices and Monetary Policy*, University of Chicago Press, pp. 371–396.

Clark, T. E. and West, K. D. (2007). Approximately Normal Tests for Equal Predictive Accuracy in Nested Models, *Journal of Econometrics* **138**: 291–311.

Coakley, J. and Fuertes, A.-M. (2006). Valuation ratios and price deviations from fundamentals, *Journal of Banking & Finance* **30**(8): 2325–2346.

Cochrane, J. H. (2008a). *Handbook of the Equity Risk Premium*, Elsevier Science, chapter 7, pp. 237–330.

Cochrane, J. H. (2008b). The Dog That Did Not Bark: A Defense of Return Predictability, *The Review of Financial Stucies* **21**: 1533–1575.

Cohen, L., Diether, K. B. and Malloy, C. J. (2007). Supply and Demand Shifts in the Shorting Market, *Journal of Finance* **LXII(5)**: 2061–2096.

Corhay, A., Rad, A. T. and Urbain, J. (1993). Common stochastic trends in European stock markets, *Economics Letters* **42**(4): 385–390.

Corsetti, G., Pesenti, P. and Roubini, N. (1999). What caused the Asian currency and financial crisis?, *Japan and the world economy* **11**(3): 305–373.

Cunado, J., Gil-Alana, L. A. and De Gracia, F. P. (2008). Stock Market Volatility in US Bull and Bear Markets., *Journal of Money, Investment and Banking* **(1)**: 24–32.

Dai, M., Zhang, Q. and Zhu, Q. (2010). Trend following trading under a regime switching model, *SIAM Journal of Financial Mathemetics* **1**: 780–810.

Davidson, R. and MacKinnon, J. (1984). Convenient Specification Tests for Logit and Probit Models, *Journal of Econometrics* **25**: 241–262.

Davidson, R. and MacKinnon, J. G. (1993). *Estimation and Inference in Econometrics*, Oxford University Press.

Davis, R. A. (2010). *Heavy Tails in Financial Time Series*, Wiley, New York.

Davutyan, N. and Parke, W. R. (1995). The Operations of Bank of England 1890-1908: A Dynamic Ordered Probit Approach, *Journal of Money, Credit and Banking* **27**: 1099–1112.

De Haan, J., Inklaar, R. and Jong-a Pin, R. (2008). Will business cycles in the euro area converge? A critical survey of empirical research, *Journal of Economic Surveys* **22**(2): 234–273.

de Jong, R. and Woutersen, T. (2011). Dynamic Time Series Binary Choice, *Econometric Theory* **27**(04): 673–702.

Demirgüç-Kunt, A. and Detragiache, E. (2000). Monitoring banking sector fragility: a multivariate logit approach, *The World Bank Economic Review* **14**(2): 287–307.

Dewachter, H. and Lyrio, M. (2006). Macro Factors and the Trem Structure of the Interest Rates, *Journal of Money, Credit and Banking* **38**: 119–240.

Dickinson, G. D. (2000). Stock market integration and macroeconomic fundamentals: An empirical analysis, 1980-95, *Applied Financial Economics* **10**(3): 261–276.

Diebold, F. X. and Mariano, R. S. (1995). Comparing Predictive Accuracy, *Journal of Business a* **13(3)**: 134–144.

Diether, K. B., Lee, K.-H. and Werner, I. M. (2009). Short-selling Strategies and Return Predictability, *The Review of Financial Studies* **22(2)**: 575–605.

Dornbusch, R. (1979). Monetary policy under exchange rate flexibility, *NBER Working Paper* (w0311).

Dornbusch, R., Branson, W., Kenen, P., Houthakker, H., Hall, R., Lawrence, R., Perry, G., Fellner, W., Brainard, W. and von Furstenburg, G. (1980). Exchange rate economics: where do we stand?, *Brookings Papers on Economic Activity* **1980**(1): 143–205.

Dornbusch, R. and Fischer, S. (1980). Exchange rates and the current account, *The American Economic Review* **70(5)**: 960–971.

Dorst, F. and Nijman, T. (1993). Temporal Aggregation of GARCH Processes, *Econometrica: Journal of the Econometric Society* pp. 909–927.

Doyle, M. B. and Faust, J. (2002). An investigation of co-movements among the growth rates of the G-7 countries, *Federal Reserve Bank of New York Bulletin* **October**: 427–437.

Doyle, M. B. and Faust, J. (2005). Breaks in the variability and comovement of G-7 economic growth, *The Review of Economics and Statistics* **87**(4): 721–740.

Dueker, M. (1999). Conditional Heteroscedasticity in Qualitative Response Models of Time Series: A Gibbs-Sampling approach to the Bank Prime Rate, *Journal of Business and Economic Statistics* **17(4)**: 466–472.

Dueker, M. J. (1997). Strenghtening the Case for the Yield Curve as a Predictor of U.S. Recessions, *Review, Federal Reserve Bank of St. Louis* **79(2)**: 41–51.

Dueker, M. J. (2002). Regime-Dependent Recession Forecast and the 2001 Recession, *Review, Federal Reserve Bank of St. Louis* **84(6)**: 29–36.

Edwards, S., Biscarri, J. G. and de Gracia, F. P. (2003a). Stock Market Cycles, Financial Liberalization, and Volatility, *Journal of International Money and Finance* **22**: 925–955.

Edwards, S., Biscarri, J. G. and de Gracia, F. P. (2003b). Stock Market Cycles, Financial Liberalization, and Volatility, *Journal of International Money and Finance* **22**: 925–955.

Eichengreen, B., Rose, A. K. and Wyplosz, C. (1995). Exchange Rate Mayhem. The Antecendents and Aftermath of Speculative Attack, *Economic Policy* **21**: 249–312.

Eichengreen, B., Rose, A. and Wyplosz, C. (1996). Contagious Currency Crises: First Tests, *Scandinavian Journal of Economics* **98**(4): 463–84.

Eichengreen, B., Rose, A., Wyplosz, C., Dumas, B. and Weber, A. (1995). Exchange market mayhem: the antecedents and aftermath of speculative attacks, *Economic policy* pp. 249–312.

Eichengreen, B., Watson, M. and Grossman, R. S. (1985). Bank Rate Policy Under Interwar Gold Standard, *Economic Journal* **95**: 725–745.

Engel, C. and West, K. (2005). Exchange rates and fundamentals, *Journal of Political Economy* **113**(3).

Engel, C. and West, K. D. (2006). Taylor Rules and the Deutschmark–Dollar Real Exchange Rate, *Journal of Money, Credit, and Banking* **38**(5): 1175–94.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity With Estimates of the Variance of United Kingdom, *Econometrica* **50(4)**: 987–1001.

Engle, R. F. (1983). Wald, Likelihood Ratio and Lagrange Multiplier Tests in Econometrics, *in* Griliches, Z. and Intrilligator, M. (ed.), *Handbook of Econometrics*, Cambridge University Press, chapter 13, pp. 775–826.

Engle, R. F. and Mezrich, J. (1996). GARCH for Groups, *Risk* **9**: 36–40.

Estrella, A. (1998). A New Measure of Fit for Equations with Dichotomous Dependent Variables, *Journal of Business & Economic Statistics* **16(2)**: 198–205.

Estrella, A. and Hardouvelis, G. A. (1991a). 'The Term Structure as a Predictor of Real Economic Activity', *The Journal of Finance* **46(2)**: 555–576.

Estrella, A. and Hardouvelis, G. A. (1991b). The Term Structure as a Predictor of Real Economic Activity, *The Journal of Finance* **46(2)**: 555–576.

Estrella, A. and Mishkin, F. S. (1997). The Predictive Power of Term Structure of Interest Rates in the Europe and the United States: Implications for the European Central Bank, *European Economic Review* **41**: 1375–1401.

Estrella, A. and Mishkin, F. S. (1998). Predicting U.S. Recessions: Financial Variables as Leading Indicators, *The Review of Economics and Statistics* **80(1)**: 45–61.

Evans, M. and Lyons, R. (1999). Order flow and exchange rate dynamics, *Technical report*, National Bureau of Economic Research.

Falcetti, E. and Tudela, M. (2006). Modelling currency crises in emerging markets: A dynamic probit model with unobserved heterogeneity and autocorrelated errors*, *Oxford Bulletin of Economics and Statistics* **68**(4): 445–471.

Faust, J., Rogers, J. and H Wright, J. (2003). Exchange rate forecasting: the errors weŠve really made, *Journal of International Economics* **60**(1): 35–59.

Flood, R. and Rose, A. K. (2001). Uncovered interest parity in crisis: The interest rate defence in the 1990s.

Francq, C., Horvath, L. and Zakoïan, J. (2011). Merits and drawbacks of variance targeting in garch models, *Journal of Financial Econometrics* **9**(4): 619–656.

Frankel, J. (1979). On the mark: A theory of floating exchange rates based on real interest differentials, *The American Economic Review* **69**: 610–622.

Frankel, J. (1982). The mystery of the multiplying marks: A modification of the monetary model, *The Review of Economics and Statistics* **64**(3): 515–519.

Frankel, J. A., Bergsten, F. and Mussa, M. L. (1994). *American Economic Policy in the 1980s*, NBER, chapter Exchange Rate Policy, pp. 293–366.

Frankel, J. A. and Rose, A. K. (1996). Currency crashes in emerging markets: An empirical treatment, *Journal of International Economics* **41**(3Ű4): 351–366.

Frankel, J. A. and Rose, A. K. (1998). The endogeneity of the optimum currency area criteria, *The Economic Journal* **108**(449): 1009–1025.

Frankle, J. A. and Rose, A. K. (1996). Currency Crashes in Emerging Markets - An Empirical Treatment, *Journal of International Economics* **41**: 351–366.

Fratzscher, M. (2002). Financial market integration in Europe: On the effects of EMU on stock markets, *International Journal of Finance and Economics* **7**(3): 165–193.

Frenkel, J. A. (1976). A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence, *the Scandinavian Journal of Economics* **78(2)**: 200–224.

Frenkel, J. A. and Mussa, M. L. (1985). *Handbook of International Economics*, Vol. 2, Elsevier, chapter 14: “Asset markets, exchange rates and the balance of payments", pp. 679–747.

Froot, K. A. (1990). Short rates and expected asset returns, *Technical report*, National Bureau of Economic Research.

Fung, W. and Hsieh, D. A. (2011). The risk in hedge fund strategies: Theory and evidence from long/short equity hedge funds , *Journal of Empirical Finance* **18**: 547–569.

Galati, G., Heath, A. and McGuire, P. (2007). Evidence of carry trade activity, *BIS Quarterly Review* **3**: 27–41.

Gerlach, P., McCauley, R. N. and Ueda, K. (2011). Currency intervention and the global portfolio balance effect: Japanese and swiss lessons, 2003-2004 and 2009-2010, *CIRJE, Faculty of Economics, University of Tokyo* **CIRJE-F-830**.

Gerrits, R.-J. and Yuce, A. (1999). Short- and long-term links among European and US stock markets, *Applied Financial Economics* **9**(1): 1–9.

Godfrey, L. (1991). *Misspecification Tests in Econometrics: The Lagrange Multiplier Principle and other Approaches*, Vol. 16 of *Ecpnometric Society Monographs*, Cambridge University Press.

Goldfajn, I. and Valdés, R. (1998). Are Currency Crises Predictable?, *European Economic Review* **42**(3-5): 873–885.

Gonzalez, L., Powell, John G. amd Shi, J. and Wilson, A. (2005). Two Centuries of Bull and Bear Market Cycles., *International Review of Economics and Finance* **14**: 469–486.

Gourieroux, C., Monfort, A., Renault, E. and Trognon, A. (1987). Generalised Residuals, *Journal of Econometrics* **32(5)**: 5–32.

Gourieroux, C., Monfort, A. and Trognon, A. (1984). Estimation and Test in Probit Models with Serial Correlation, *Alternative Approaches to Time Series Analysis*, Facultes Universitaires Saint-Louis, Brussels.

Gourieroux, C., Monfort, A. and Trognon, A. (1985). A General Approach to Serial Correlation, *Econometric Theory* **01(3)**: 315–340.

Gourinchas, P.-O. and Tornell, A. (2004). Exchange rate puzzles and distorted beliefs, *Journal of International Economics* **64**(2): 303–333.

Goyal, A. and Welch, I. (2003). Predicting the Equity Premium with Dividend Ratios, *Management Science* **49(5)**: 639–654.

Goyal, A. and Welch, I. (2008). A Comprehensive Look at the Empirical Performance of Premium Prediction, *Review of Financial Studies* **21(4)**: 1455–1508.

Granger, C. W., Huangb, B.-N. and Yang, C.-W. (2000). A bivariate causality between stock prices and exchange rates: evidence from recent asianflu, *The Quarterly Review of Economics and Finance* **40**(3): 337–354.

Greene, W. H. (2008). *Econometric analysis*, New Jersey: Pearson Education Inc.

Guidolin, M. and Timmermann, A. (2005). Economic Implications of Bull and Bear Regimes in UK Stock and Bond Returns, *The Economic Journal* **115**: 111–143.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and Business Cycles, *Econometrica* **57**: 357–384.

Hamilton, J. D. and Lin, G. (1996). Stock Market Volatility and the Business Cycles., *Journal of Applied Econometrics* **11**: 573–593.

Han, Y., Yang, K. and Zhou, G. (2011). A New Anomaly: The Cross-sectional Profitability of Technical Analysis, *SSRN* .
URL: *http://ssrn.com/abstract=1656460*

Hansen, P. and Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH (1, 1)?, *Journal of Applied Econometrics* **20**(7): 873–889.

Harding, D. and Pagan, A. R. (2006). Synchronization of cycles, *Journal of Econometrics* **132**(1): 59–79.

Hartmann, P., Straetmans, S. and de Vries, C. G. (2006). Banking system stability: A cross-atlantic perspective, *The risks of financial institutions*, Carey, M. and Stulz, R. (Eds.), The University of Chicago Press (Chicago and London), pp. 133–193.

Harvey, A. (1976). Estimating regression models with multiplicative heteroscedasticity, *Econometrica: Journal of the Econometric Society* pp. 461–465.

Harvey, A., Ruiz, E. and Sentana, E. (1992). Unobserved component time series models with ARCH disturbances, *Journal of Econometrics* **52**(1): 129–157.

Hausman, J. A., Lo, A. W. and McKinlay, A. C. (1992). An Ordered Probit Analysis of Transaction Stock Prices, *Journal of Financial Economics* **31**: 319–379.

Haynes, S. and Stone, J. (1981). On the mark: Comment, *The American Economic Review* **71**: 1060–1067.

Helbling, T. and Bayoumi, T. (2003). Are they all in the same boat? The 2000-2001 growth slowdown and the G-7 business cycle linkages, *IMF Working Paper No. 46* (4).

Hodrick, R. J. and Prescott, E. C. (1997). Post-war US business cycles: An empirical investigation, *Journal of Money, Credit and Banking* **29**(1): 1–16.

Hsieh, D. (1989). Testing for nonlinear dependence in daily foreign exchange rates, *Journal of Business* pp. 339–368.

Hutchison, M. and Noy, I. (2005). How bad are twins? output costs of currency and banking crises, *Journal of Money, Credit, and Banking* **37**(4): 725–752.

Imbs, J. (2004). Trade, finance, specialization, and synchronization, *The Review of Economics and Statistics* **86**(3): 723–734.

Jevons, W. S. (1909). The Solar Period and the Price of Corn, *in* H. S. Foxwell (ed.), *Investigations in Currency and Finance*, MaMillan, London, pp. 194–205.

Jorda, O. and Taylor, M. A. (2012). The carry trade and fundamentals: Nothing to fear but feer itself, *Journal of International Economics* **88**: 74–90.

Kalemli-Ozcan, S., Sorensen, E. B. and Yosha, O. (2001). Economic integration, industrial specialization, and the asymmetry of macroeonomic fluctuations, *Journal of International Econmics* **55**(1): 107–137.

Kaminsky, G., Lizondo, S. and Reinhart, C. (1998). Leading indicators of currency crises, *Staff Papers-International Monetary Fund* pp. 1–48.

Kaminsky, G. and Reinhart, C. (1999). The twin crises: the causes of banking and balance-of-payments problems, *American Economic Review* pp. 473–500.

Kauppi, H. and Saikkonen, P. (2008). Predicting U.S. Recessions With Dynamic Binary Response Models, *The Review of Economics and Statistics* **90(4)**: 777–791.

Kilian, L. and Taylor, M. P. (2003). Why is it so difficult to beat the random walk forecast of exchange rates?, *Journal of International Economics* **60**(1): 85 – 107.

Kim, S. J., Moshirian, F. and Wu, E. (2005). Dynamic stock market integration driven by the European Monetary Union: An empirical analysis, *Journal of Banking and Finance* **29**(10): 2475–2502.

King, M. R. and Rime, D. (2010). The $4 Trillion Question: What Explains FX Growth since the 2007 Survey?, *BIS Quarterly Review* **December**.

Knif, J. and Pynnonen, S. (1999). Local and global price memory of international stock markets, *Journal of International Financial Markets, Institutions and Money* **9**(2): 129–147.

Kose, M. A., Otrok, C. and Whiteman, H. C. (2008). Understanding the evolution of world business cycles, *Journal of International Economics* **75**(1): 110–130.

Kouri, P. J. K. (1976). The exchange rate and the balance of payments in the short run and in the long run: A monetary approach, *The Scandinavian Journal of Economics* **78(2)**: 280–304.

Krugman, P. (1979). A Model of Balance of Payments Crises, *Journal of Money, Credit and Banking* pp. 311–325.

Krugman, P. (1991). *Geography and trade*, The MIT Press.

Krugman, P. and Obstfeld, M. (2009). *International economics: Theory and policy*, Boston, Massachusetts: Pearson, Addison-Wesley.

Krugman, P. R., Obstfeld, M. and Melitz, M. J. (2012). *International Economics: Theory and Policy*, 9th edn, Addison-Wesley (Pearson Education, Inc.).

Krugman, P. and Venables, A. (1993). Integration, specialization, and adjustment, *NBER Working Paper No. 4559* .

Kumar, M., Moorthy, U. and Perraudin, W. (2003). Predicting emerging market currency crashes, *Journal of Empirical Finance* **10**(4): 427–454.

Lee, C. I., Gleason, K. C. and Mathur, I. (2001). Trading rule profits in latin american currency spot rates, *International Review of Financial Analysis* **10**(2): 135 – 156.

Lee, J. (1991). A Lagrange multiplier test for GARCH models, *Economics Letters* **37**(3): 265–271.

Levich, R. M. and Thomas III, L. R. (1993). The significance of technical trading-rule profits in the foreign exchange market: a bootstrap approach, *Journal of International Money and Finance* **12**(5): 451 – 474.

Lui, Y.-H. and Mole, D. (1998). The use of fundamental and technical analyses by foreign exchange dealers: Hong kong evidence, *Journal of International Money and Finance* **17**(3): 535 – 545.

MacDonald, R. and Taylor, M. (1989). Foreign exchange market efficiency and cointegration: Some evidence from the recent float, *Economics Letters* **29**(1): 63–68.

MacDonald, R. and Young, R. (1986). Decision rules, expectations and efficiency in two foreign exchange markets, *De economist* **134**(1): 42–60.

Maheu, J. M. and McCurdy, Thomas, H. (2000). Identifying Bulls and Bear Markets in Stock Returns, *Journal of Business & Econoic Statistics* **18(1)**: 100–112.

Mark, N. C. (1995). Exchange Rates and Fundamentals: Evidence on Long-horizon Predictability, *The American Economic Review* pp. 201–218.

Mark, N. and Sul, D. (2001). Nominal exchange rates and monetary fundamentals: evidence from a small post-bretton woods panel, *Journal of International Economics* **53**(1): 29–52.

Marquering, W. and Verbeek, M. (2004). The Economic Value of Predicting Stock Index Returns and Volatility, *Journal of Financial and Quantitative Analysis* **39(2)**: 407–429.

McCauley, R. N. and McGuire, P. (2009). Dollar appreciation in 2008: safe haven, carry trade, dollar shoshort and overhedging, *BIS Quarterly Review* **December**: 85–93.

McCullough, B. and Renfro, C. (1998). Benchmarks and software standards: A case study of garch procedures, *Journal of Economic and Social Measurement* **25**(2): 59–72.

McKinnon, R. and Pill, H. (1997). Credible economic liberalizations and over-borrowing, *The American Economic Review* **87**(2): 189–193.

Meese, R. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample?, *Journal of international economics* **14**(1): 3–24.

Melvin, M. and Taylor, M. P. (2009). The crisis in the foreign exchange market, *Journal of International Money and Finance* **28**(8): 1317 – 1330.

Menkhoff, L., Sarno, L., Schmeling, M. and Schrimpf, A. (2011). Carry Trades and Global Foreign Exchange Volatility.

Menkhoff, L. and Taylor, M. P. (2007). The Obstinate Passion of Foreign Exchange Professionals: Technical Analysis, *Journal of Economic Literature* **45**(4): 936–972.

Micco, A., Stein, E. and Ordonez, G. (2003). The currency union effect on trade: Early evidence from EMU, *Economic Policy* **18**(37): 315–356.

Mikosch, T. and Starica, C. (2000). Limit theory for the sample autocorrelations and extremes of a garch (1, 1) process, *Annals of Statistics* pp. 1427–1451.

Minsky, H. P. (1977). The Financial Instability Hypothesis: An Interpretation Of Keynes And An Alternative To 'Standard' Theory, *Nebraska Journal of Economics and Business* **16(1)**: 5–16.

Mishkin, F. S. (1990). What does the term structure tell us about future inflation?, *Journal of monetary economics* **25**(1): 77–95.

Mishkin, F. S. (1991). The information in the longer maturity term structure about future inflation, *Technical report*, National Bureau of Economic Research.

Mishkin, F. S. and Eakins, S. G. (2012). *Financial Markets and Institutions*, Pearson.

Molodtsova, T. and Papell, D. (2009). Out-of-sample exchange rate predictability with taylor rule fundamentals, *Journal of International Economics* **77**(2): 167–180.

Morley, B. (2002). Exchange rates and stock prices: implications for european divergence, *Journal of Policy Modeling* **24**: 523–526.

Mundell, A. R. (1961). A theory of optimum currency areas, *The American Economic Review* **51**(4): 657–665.

Mussa, M. (1976). The exchange rate, the balance of payments and monetary and fiscal policy under a regime of controlled floating, *The Scandinavian Journal of Economics* pp. 229–248.

Mussa, M. (1979). *Empirical regularities in the Behaviour of Exchange Rates and Theories of Foreign Exchange Market*, pp. 9–57.

Neftci, S. N. (1984). Are Economic Time Series Asymmetric over Business Cycles?, *Journal of Political Economy* **92(2)**: 307–328.

Nelson, C. R. and Siegel, A. F. (1987). Parsimonious Modeling of Yield Curves, *Journal of Business* **60**: 473–489.

Newey, W. and West, K. (1987). A Simple Positive Semi-definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* **55**: 703–708.

Newman, V., Potter, C. and Wright, M. (2011). Foreign Exchange Market Intervetions, *RBA Bulletin* **December**: 67–76.

Nyberg, H. (2011). Forecasting the direction of the US stock market with dynamic bianry Probit model, *International Journal of Forecasting* **27**: 561–578.

Obstfeld, M. (1996). Models of currency crises with self-fulfilling features, *European Economic Review* **40**(3): 1037–1047.

Obstfeld, M. and Rogoff, K. (1996). *Foundations of International Macroeconomics*, MIT Press, Cambrdige MA.

Pagan, A. R. and Sossounov, K. A. (2003). A Simple Framework for Analysis of Bull and Bear Markets, *Journal of Applied Econometrics* **18**: 23–46.

Papell, D. and Molodtsova, T. (2012). Taylor rule exchange rate forecasting during the financial crisis, *NBER International Seminar on Macroeconomics 2012*, University of Chicago Pres.

Perez-Quiros, G. and Timmermann, A. (2000). Firm size and cyclical variations in stock returns, *The Journal of Finance* **55**(3): 1229–1262.

Pesaran, H. M. and Timmermann, A. (1994). Forecasting Stock Returns: An Examination of Stock Market Trading in the Presence of Tracnsaction Costs, *Journal of Forecasting* **13(4)**: 335–367.

Pesaran, H. M. and Timmermann, A. (1995). Predictability of Stock returns: Robustness and Economic significance, *Journal of Finance* **50(4)**: 1201–1228.

Pollin, R. and Heintz, J. (2011). Transaction Costs, Trading Elasticities and the Revenue Potential of Financial Transaction Tax for the United States, *Research Brief - Political Economy Research Institute, University of Massachusetts, Amherst* .

Poole, W. (1967). Speculative prices as random walks: an analysis of ten time series of flexible exchange rates, *Southern Economic Journal* pp. 468–478.

Ranaldo, A. and Söderlind, P. (2010). Safe Haven Currencies, *Review of Finance* **14**(3): 385–407.

Rapach, D. E., Wohar, M. E. and Rangvid, J. (2005). Macro-variables and International Stock Return Predictability, *International Journal of Forecasting* **21**: 137–166.

Rapach, D. and Wohar, M. (2002). Testing the monetary model of exchange rate determination: new evidence from a century of data, *Journal of International Economics* **58**(2): 359–385.

Resnick, B. G. and Shoesmith, G. L. (2002). Using the Yield Curve to Time the Stock Market, *Financial Analysts Journal* **58(3)**: 82–90.

Rich, G. (1990). Exchange-rate management under floating exchange rates: A skeptical swiss view, *Journal of Banking & Finance* **14**(5): 993–1021.

Roll, R. (1979). *Violations of purchasing power parity and their implications for international commodity markets*, Ballinger, Cambridge, M. A.

Rudebusch, G. D. and Wu, T. (2008). A Macro-Finance Model of Term Structure, Monetary Policy and the Economy, *Economic Journal* **118**: 906–926.

Sarno, L. (2005). Viewpoint: Towards a solution to the puzzles in exchange rate economics: where do we stand?, *Canadian Journal of Economics/Revue canadienne d'économique* **38**(3): 673–708.

Schiller, R. J. (2000). *The Irrational Exuberance*, Wiley Online Library.

Shen, P. (2003). Market Timing Strategies that Worked: Based on E/P ratio of the S&P 500 and interest rates., *Journal of Portfolio Management* **29(2)**: 57–68.

SNB (2009). Monetary policy assessment of 12 march 2009, *Swiss National Bank* .

Soros, G. (2010). *The Soros Lectures: At the Central European University*, Public Affairs Store.

Spronk, R., Verschoor, W. F. and Zwinkels, R. C. (2013). Carry trade and foreign exchange rate puzzles, *European Economic Review* .

Stock, J. and Watson, M. (1993). *Business Cycles, Indicators, and Forecasting*, University of Chicago Press.

Taylor, M. and Allen, H. (1992). The use of technical analysis in the foreign exchange market, *Journal of international Money and Finance* **11**(3): 304–314.

Taylor, P. M. and Tonks, I. (1989). The internationalization of stock markets and the abolition of U.K. exchange control, *The Review of Economics and Statistics* **71**(2): 332–336.

Taylor, S. J. (1986). *Modelling financial time series*, Vol. 113, World Scientific.

Walti, S. (2011). Stock market synchronization and monetary integration, *Journal of International Money and Finance* **30**(1): 96–110.

Wang, J. and Wu, J. (2012). The taylor rule and forecast intervals for exchange rates, *Journal of Money, Credit and Banking* **44**(1): 103–144.

Watson, M. W. (1994). Business-cycle durations and postwar stabilization of the us economy, *The American Economic Review* pp. 24–46.

White, H. (1982). Maximum Likelihood Estimation of Misspecified Models, *Econometrica* **50(1)**: 1–25.

Nederlandse Samenvatting

Deze proefschrift bestaat uit vier individuele essays over zakelijke en financile cycli. We pogen zakelijke en financile cycli te voorspellen door middel van de dynamische extensies van binaire-keuzemodellen, rekening houdend met de nawerking, het clustereffect en de heteroscedasticiteit. In hoofdstuk 2, met name, doen we voorspellingen over de 'bear' fasen op de Amerikaanse aandelenmarkt. We concluderen dat het gunstig is rekening te houden met de gestileerde feiten van cyclische reeksen bij het voorspellen van deze cycli. Waar cyclusvoorspellingen op de vermogensmarkt (bear of bull) beleidsmakers en beurswaakhonden kunnen helpen proactief maatregelen te nemen om excessieve prijzen voor goederen te beperken, blijken ze evenzeer nuttig voor investeerders om het juiste handelsmoment te bepalen, actief te handelen en de passieve strategie van kopen en vasthouden te verbreken.

In hoofdstuk 3 richten we ons op een andere klasse financile producten en bekijken we de cycli van bilaterale wisselkoersen. Hier bestuderen we de cycli van 's werelds zes belangrijkste valuta's die worden beschouwd als 'reservevaluta', 'funding valuta; of 'investeringsvaluta'. We abstraheren cyclische perioden van waardestijging en daling niet-parametrisch en proberen deze perioden n tot 24 maanden vooruit te voorspellen op basis van vijf risicofactoren. Grof gezegd zijn deze risicofactoren schendingen van UIP, RPPP, pseudo-vermogenspariteit, liquiditeitsdruk en term spread. Met deze risicofactoren vertonen wisselkoersen daadwerkelijk cyclische variaties met perioden van waardestijging volgend op waardedaling en vice versa. Dit impliceert dat beleidsmakers deze signalen kunnen gebruiken om valuta-afwijkingen tegen te gaan en de concurrentiepositie te herstellen. Daarnaast maakt de carry trade over het algemeen het leeuwendeel uit van de handelsactiviteit op de forexmarkt. Negatieve ontwikkelingen in de doel- of funding valuta's kan de cumulatieve winst van handelaars eroderen. Daarom kunnen voorziene periodes van waardestijging en -daling carry traders helpen hun verliezen te beperken doordat ze hun portefeuilles kunnen aanpassen.

In het voorlaatste hoofdstuk, hoofdstuk 4, bestuderen we bilaterale synchronisatie van rele en financile cycli tussen negen landen in de eurozone. Op basis van het probit-framework, zien we sterke synchronisatie van zakelijke en financile cycli tussen landen. Dit blijkt uit statistisch en economisch significante marge-effecten binnen het probit-framework. Bovendien overschaduwt financile synchronisatie de synchronisatie van zakelijke cycli in de eurozone, met name sinds de invoering van de gezamenlijke munt: waar de euro-sample samenvalt met een sterke toename van financile synchronisatie, blijft de synchronisatie van zakelijke cycli vrijwel stabiel. Voor sommige landenparen vinden we zelfs aanwijzingen van het 'ontkoppelen' van zakelijke cycli, maar de meerderheid van de marginale effecten verandert weinig door de tijd. Rekening houden met endogeniteit heeft geen fundamentele invloed op onze resultaten. Onze resultaten suggereren dat monetaire integratie meer financile integratie heeft veroorzaakt, maar de impact van monetaire integratie op de synchronisatie van zakelijke cycli blijft beperkt of leidt in een aantal gevallen zelfs tot een 'ontkoppeling' van de cycli van perifere landen in relatie tot die van kernlanden. De eerste observatie valt in lijn met veel gehoorde oproep voor meer internationale 'macro-prudential' regulering, terwijl de tweede observatie de economen ondersteunt die benadrukten dat de architectuur van eurozone onaf was en dat de condities voor een optimale valuta-zone nog niet zijn vervuld.

Uiteindelijk behandelen we in hoofdstuk 5 het tot nu toe verwaarloosde aspect van heteroscedasticiteit in binaire modeling frameworks. We bouwen voort op de observatie dat serile afhankelijkheid en volatiliteitsclustering, onder meer, leidt tot conditionele heteroscedasticiteit in verstoringen van het model (Engle, 1982). We stellen een aanpassing in BCMs voor voor conditionele heteroscedasticiteit in de lijn van Engle (1982) and Bollerslev (1986). Met name stellen we een GARCH-achtige aanpassing voor voor de covariantie van fouten op korte termijn, terwijl we de onconditionele variantie gekoppeld laten aan unity om identificatieredenen. Resultaten van simulaties en empirische applicaties aangaande het voorspelling van Amerikaanse zakelijke en financile cycli op basis van term spread bevestigen het nut van heteroscedastische aanpassingen. Bovendien stellen we LM-tests voor voor ARCH-effecten in BCMs. Een van de voorgestelde LM-tests is gebaseerd op de verwachte informatiematrix en vertoont een kleinere vertekening en een groter onderscheidend vermogen.

Biography

Jameel Ahmed was born on September 04, 1973 in Khairpur Mirs, Pakistan. He studied masters in business administration (MBA), specialization Finance, at Quaid-e-Azam University, Islamabad, Pakistan. He joined State Bank of Pakistan (SBP), the central bank, as a regulating officer in 1999 and is currently a deputy director at Monetary Policy Department. Proceeding on study leave from SBP, he pursued and completed in 2008 a masters in Economics and Business (Financial Economics) at Erasmus University, Rotterdam, The Netherlands.

In September 2008, Jameel joined Maastricht University's MPhil (Economic and Financial Research) program and later started his doctoral studies as an external researcher in April 2010, jointly at departments of Economics and Finance. His area of research is broadly the Financial Economics, focusing on the prediction and synchronization of business and financial cycles. During his doctoral studies, he also served as a part-time lecturer at Department of Finance and was involved in teaching/tutoring Finance, Behavioral Finance, Shareholder Value and Market Based Assets and Accounting. His work has been presented at various international conferences including the International Finance and Banking Conference, Valencia, Spain (2012), the Surrey-Fordham Conference, Surrey, UK (2013).